A Learning and Control Perspective for Microfinance

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Abstract

Microfinance, despite its significant potential for poverty reduction, is facing sustainability hardships due to high default rates. Although many methods in regular finance can estimate credit scores and default probabilities, these methods are not directly applicable to microfinance due to the following unique characteristics: a) under-explored (developing) areas such as rural Africa do not have sufficient prior loan data for microfinance institutions (MFIs) to establish a credit scoring system; b) microfinance applicants may have difficulty providing sufficient information for MFIs to accurately predict default probabilities; and c) many MFIs use group liability (instead of collateral) to secure repayment. Here, we present a novel control-theoretic model of microfinance that accounts for these characteristics. We construct an algorithm to learn microfinance decision policies that achieve financial inclusion, fairness, social welfare, and sustainability. We characterize the convergence conditions to Pareto-optimum and the convergence speeds. We demonstrate, in numerous real and synthetic datasets, that the proposed method accounts for the complexities induced by group liability to produce robust decisions before sufficient loans are given to establish credit scoring systems and for applicants whose default probability cannot be accurately estimated due to missing information. To the best of our knowledge, this paper is the first to connect microfinance and control theory. We envision that the connection will enable safe learning and control techniques to help modernize microfinance and alleviate poverty.

1 Introduction

Potential and challenges in microfinance. Microfinance is a category of financial services that gives small loans to low-income people who may not have access to or be eligible for conventional finance (Yunus 2007; Armendáriz and Morduch 2010; Kamanza 2014). Microfinance has demonstrated potential for poverty reduction, financial inclusion, and economic development (Schreiner 2001; Mersland and Strøm 2010). Despite the proven potential, microfinance has experienced several hardships, primarily due to increasing loan default rates (Nawai and Shariff 2012; Addae-Korankye 2014).

Although there are many lending strategies and risk control methods in regular finance, they cannot be directly applied to microfinance for the following reasons. First, most existing methods use credit scores to predict the loan default probability when making lending decisions (Ala'raj and Abbod 2016; Shi et al. 2019; Ampountolas et al. 2021). However, under-explored (developing) areas without prior loan histories or proper financial systems have insufficient data to establish such credit-scoring procedures. Second, due to the lack of proper state mechanisms, it is difficult for some applicants to provide sufficient information for estimating their credit scores and default probability accurately[1]. Third, regular loans are given to individuals with collateral, whereas microfinance often uses group liability, without collateral, to secure repayment (Lehner 2009; Kodongo and Kendi 2013; Haldar and Stiglitz 2016). Group liability can improve the repayment rate by incentivizing members to look after each other, but it has the pitfalls of inducing defaults for borrowers who otherwise have the ability to repay. However, because the approaches for granting regular loans do not sufficiently account for the complexities of group liability, group loans have tended to result in greater default rates (Nandhi 2012; Allen 2016). Fourth, there is increasing evidence that loan approval algorithms based on black-box machine learning techniques may be biased and discriminatory against minorities (Zliobaite 2015; Corbett-Davies and Goel 2018). Such biases are particularly problematic in fulfilling the objectives of microfinance to provide more opportunities for disadvantaged populations and underdeveloped regions. The complexities of having multiple such populations/regions also imposed additional challenges in allocating microfinance resources to balance different fairness/inclusion objectives. Due to the lack of methodologies that can systematically balance the risks, fairness, and multi-faceted objectives of microfinance, microfinance has relied heavily on the judgment of loan officers. Such operations have sometimes resulted in decisions that let Portfolio at Risk (PAR)[2] exceed a level that is sustainable for continuing microfinance operations[3] (Yimga 2016; Huo and Fu 2017; Chikalipah 2018).

Our focus and contributions. There is an urgent need to modernize microfinance by establishing models and algorithms that account for the aforementioned characteristics and challenges. In this paper, we establish a novel microfinance model and propose an algorithm for learning loan

[1] For example, it may be costly for some applicants in Africa to obtain proof of residence.

[2] PAR is defined as the percentage of overdue loans.

[3] World Bank suggested 5% as the upper bound of PAR for sustainability (Ledgerwood, Earne, and Nelson 2013).

approval policies. We summarize the features of the proposed techniques below.

1. Our methods can make robust decisions *before* enough loans are given to accurately estimate the default probability and credit scores (Figure 2b) by directly learning the optimal policy parameters without the intermediate step of default probability estimation.
2. Our methods degrade more gracefully for increasing levels of missing information in the applications (Figure 2a) and exploit the potential of group liability while avoiding its pitfalls (Figure 2e). The microfinance model accounts for missing information and group liability, and policy learning processes converge to optimal policy parameters in the presence of both.
3. Our methods can systematically optimize competing objectives such as risks, socio-economic impacts, and active and passive fairness among different groups (Figures 2i and 2k). The prioritization among different objectives can be specified in the utility function, and the policy has an interpretable structure that informs which factors contributed positively/negatively to applicant approvals.

To the best of our knowledge, this paper is the first to use control-theoretic techniques to learn microfinance policy parameters without relying on credit scores directly. Our presentation of microfinance models as a *control problem* opens the door to using modern control and learning theory to modernize microfinance, which in turn helps to achieve 8 of 17 Sustainable Development Goals adopted by the United Nations.

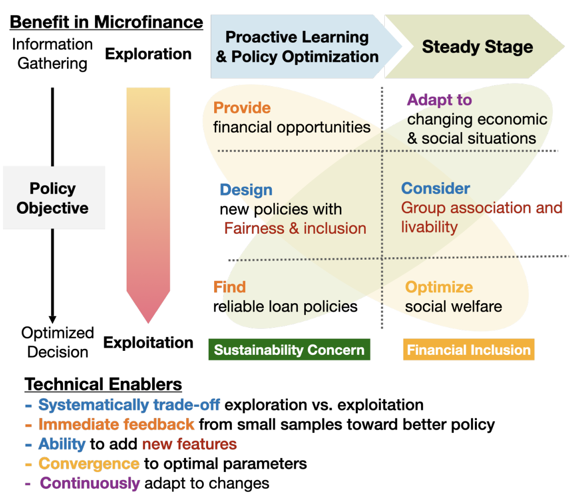


Figure 1: Features of the proposed algorithm and their technical enablers.

2 Problem Statement

In this section, we introduce a novel microfinance model and define the design objectives of the microfinance decision policy.

Microfinance model: A microfinance application can be modeled by application properties, an MFI's decision, and outcomes. The MFI receives applications from individuals or groups of applicants. An application is parameterized by the group size M ($M = 1$ for individual applicants), intrinsic features that govern default probability $S \in \mathcal{S}$, and the MFI's accessible information $\hat{S} \in \hat{\mathcal{S}}$. When some information in S is unavailable, it corresponds to the empty value $\emptyset$ entries in $\hat{S}$. The set of the available information in $\hat{S}$ is denoted by $U(\hat{S}) = \{j : \hat{S}[j] \neq \emptyset\}$. The MFI's lending decision is denoted by a random variable A:

$$A = \begin{cases} 1 & \text{for approval,} \\ 0 & \text{for rejection.} \end{cases} \tag{1}$$

The MFI's approval/reject probability, $\mathbb{P}(A \mid \hat{S}, M)$, for a certain application is based on the lending policy π_Z, *i.e.*,

$$\mathbb{P}(A \mid \hat{S}, M) = \pi_Z(\hat{S}, M, A). \tag{2}$$

Here, π_Z is controlled by policy parameter Z, defined later in (12).

As the amount of loans given out by MFIs is normally small for each individual, we assume the amount of loan and its interest rate are identical among members within the group and, without loss of generality, are set as 1 and r, respectively. Thus, an approved application of group size M receives a loan of size M and must return the principal and interest of $M \cdot (r + 1)$ at the end of the lending period, where the loan liability is imposed on the whole group. The outcome of the loan (the ability of the applicant to return) is given by

$$B = \begin{cases} 1 & \text{for return,} \\ 0 & \text{for default.} \end{cases} \tag{3}$$

We assume S is independently drawn from some underlying population feature distribution $\mathbb{P}(S)$; $\hat{S}$ is determined based on how features are reflected in the accessible information $\mathbb{P}(\hat{S} \mid S)$; and the outcome of the application is governed by $\mathbb{P}(B \mid S, M) = \mathbb{P}(B \mid S, \hat{S}, M)$, which does not depend on $\hat{S}$ given S and M.

Microfinance decision criteria: We consider the setting that MFI provides loans at each lending period, indexed by $t \in \{1, 2, \cdots, T\}$, and continuously learns more optimal policies over the time horizon T as follows. At the beginning of each lending period, the MFI receives N_t financing applications, indexed by $i \in \mathcal{N}_t = \{1, 2, \cdots, N_t\}$. Application i has group size $m_{i,t} \overset{i.i.d}{\sim} \mathbb{P}(M = m_{i,t})$, unobserved underlying features $s_{i,t} \overset{i.i.d}{\sim} \mathbb{P}(S)$, and accessible information $\hat{s}_{i,t} \sim \mathbb{P}(\hat{S} \mid S = s_{i,t})$. The MFI uses policy π_{z_t} to decide on MFI's action $a_{i,t} \sim \mathbb{P}(A \mid \hat{S} = \hat{s}_{i,t},\ M = m_{i,t}) = \pi_{z_t}(\hat{s}_{i,t}, m_{i,t}, a_{i,t})$. At the end of the lending period, the MFI observes the loan outcome $b_{i,t} \overset{i.i.d}{\sim} \mathbb{P}(B \mid S = s_{i,t},\ M = m_{i,t})$ and learns (updates) the policy parameter to z_{t+1}, which is to be used in the next lending period.[4]

[4]Throughout the paper, we use lower case letters $s_{i,t}, \hat{s}_{i,t}, a_{i,t}, b_{i,t}$, to represent the specific realization of the random variables $S, \hat{S}, A, B$ that are associated with application i at time t; and z_t represents the learned policy parameter at time t.

Microfinance has multifaceted (non-mutually exclusive) objectives such as financial inclusion, fairness, social and economic impact, and sustainability. These objectives are captured by a utility function $\mathcal{R}(\{\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i\in\mathcal{N}_t})$ of all applications $i \in \mathcal{N}_t$ (see Section 5 for the empirical results for all these objectives).

We let $V(z_t) = \mathbb{E}(\mathcal{R}(\{\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i\in\mathcal{N}_t}))$ as the expected utility of the lending period t with policy π_{z_t} and control parameter z_t. We then consider two types of utility rewards: decomposable and non-decomposable.

Case 1: Decomposable rewards. In this scenario, we assume all the features are non-biased and will not introduce any discrimination into the decision-making process. Therefore, the total reward at time t *can* be decomposed as the sum of individual rewards, *i.e.*,

$$\mathcal{R}\left(\{s_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i\in\mathcal{N}_t}\right) = \frac{1}{N_t}\sum_{i=1}^{N_t} R\left(s_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\right), \tag{4}$$

for some function R.

Case 2: Non-decomposable rewards. In this scenario, we assume the total reward *cannot* be decomposed as the sum of individual rewards. This case is particularly useful when we design reward functions that account for fairness among different demographics.

Accounting for fairness. Microfinance decisions should be fair and avoid discriminating against certain populations or regions. We consider the following three types of fairness.

Type 1 (Independence): Type 1 fairness ensures that certain feature attributes will not affect the outcome of loan approval. For example, it can capture the issues raised by (Martinez and Kirchner 2021) where people with almost the same attributes except their race have largely different approval rates. If any two applications i_1, i_2 have identical information except attribute ξ, then their approval probabilities are also identical, *i.e.*,

$$\mathbb{P}\left(a_{i_1,t} = 1 \mid \hat{s}_\iota, \hat{s}_\xi, m_{i,t}\right) = \mathbb{P}\left(a_{i_2,t} = 1 \mid \hat{s}_\iota, \hat{s}_{\xi'}, m_{i,t}\right), \tag{5}$$

where $\hat{s}_\iota$ represents identical information for both applications. One way to deal with the type 1 fairness issue is to remove some features from the accessible information $\hat{S}$. For example, features such as gender, race, and ethnicity should be removed before learning the lending decision. However, even though straightforward bias features are deleted, other features may be correlated with those bias features and lead to discrimination. For example, even though we remove the gender feature, other features such as occupation may be correlated with gender. This motivates us to consider the following two types of active fairness.

Type 2 (Outcome fairness): Type 2 fairness actively sets a target approval rate $\Pi^*(\xi)$ for the desire approval rate for applications with attribute ξ. For example, the loan approval policy has a target of at least Π^* approval rate for female applicants as a criterion for gender equality. For type 2 fairness, we want to have

$$\mathbb{P}\left(a_{i,t} = 1 \mid i \in \mathcal{N}_{t,\xi}\right) \geq \Pi^*(\xi), \forall \xi \in \Xi, \tag{6}$$

where $\Xi = \{\xi, \xi', \cdots\}$ is the set of attributes for which we have a target approval rate and $\mathcal{N}_{t,\xi} = \{i \in \mathbb{N} : i \in \mathcal{N}_t, \hat{S}_\xi \in \hat{S}\}$ is the set of applications with attribute ξ at time t. To achieve type 2 fairness, we can reformulate the reward function as

$$\mathcal{R}\left(\{s_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i\in\mathcal{N}_t}\right) = \text{other objectives} - \mathcal{F}_2 \cdot \sum_{\xi\in\Xi}\left(\Pi^*(\xi) - g_{\xi,t}\right)_+, \tag{7}$$

where $\mathcal{F}_2 \in \mathbb{R}_+$ is a weighing factor, $\bar{a}_{\xi,t} = \frac{1}{|\mathcal{N}_{t,\xi}|}\sum_{i\in\mathcal{N}_{t,\xi}} a_{i,t}$ be the current approval rate for applications with ξ, and we use the notion $(x)_+$ to be $\max(0, x)$. A larger $\mathcal{F}_2$ value indicates more emphasis is placed upon type 2 fairness.

Type 3 (Statistical parity): Type 3 fairness enforces fairness among applications with different attributes, *i.e.*, if we would like to have type 3 fairness among applications with attributes ξ and ξ', then we should have

$$\mathbb{P}\left(a_i = 1 \mid i \in G_\xi\right) \approx \mathbb{P}\left(a_i = 1 \mid i \in G_{\xi'}\right). \tag{8}$$

To enforce type 3 fairness, we can adjust our reward as,

$$\mathcal{R}\left(\{s_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i\in\mathcal{N}_t}\right) = \text{other objectives} - \mathcal{F}_3 \cdot \|g_{\xi,t} - g_{\xi',t}\|, \tag{9}$$

where a larger value of $\mathcal{F}_3$ indicates more emphasis is put on type 3 fairness.

Policy learning objectives. The learning objective for the policy is twofold. The first goal is to converge to the optimal policy parameter,

$$z^* = \arg\max_z V(z). \tag{10}$$

The second goal is to converge to the optimal policy fast with low policy exploration cost, quantified by

$$V(z^*) - \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} V(z_t)\right]. \tag{11}$$

The expectation in $V(z)$ is taken over the probability measure involving group liability and missing information. Thus, by construction, the optimization of (10) and (11) also accounts for the above-mentioned challenges. Additionally, we take interpretability into the design consideration by imposing structures in policy π_z so that the parameter z informs how much each entry of available information $\hat{S}$ and the group size M have contributed toward approvals or denials.

3 Proposed Algorithm

Here, we will introduce novel learning techniques that can produce optimal and fair microfinance decisions when credit-scoring systems cannot function properly due to the scarcity of prior loan data and the uncertainty of missing data. The theoretical properties of the proposed techniques are derived in the next section.

We propose a lending policy π_z parameterized by

$$z = [\phi^\intercal, \epsilon^\intercal, \gamma^\intercal]^\intercal \in \mathcal{Z} \subset \mathbb{R}^{2n+|\Xi|}. \tag{12}$$

We update the policy parameter z_t according to:

$$\hat{z}_{t+1} = z_t + \alpha_t F_{z_t}, \tag{13}$$

$$z_{t+1} = \operatorname*{proj}_{\mathcal{Z}}(\hat{z}_{t+1}). \tag{14}$$

The value of $\alpha_t > 0$ is the step size to update the parameters at lending period t. The choice of α_t is crucial for the convergence and learning speed of the algorithm and is studied theoretically in Corollary 4.2 and empirically in Appendix D. To make sure that the updated parameters stay in the allowable domain $\mathcal{Z}$, step (14) projects $\hat{z}_{t+1}$ onto domain $\mathcal{Z}$. We then consider the decision policy

$$\pi_z(\hat{s}, m, a) = L(q). \tag{15}$$

$L(q)$ can be any continuously-differentiable monotonically-increasing function of q that map the domain of q to $(0, 1)$. For example, we consider the following choice of $L(q)$:

$$L(q) = \frac{2\exp(q)}{1 + \exp(q)} - 1. \tag{16}$$

Here, $L(q)$ can be thought of as the activation function of the neural networks. However, the traditional approaches to directly employing the neural networks for the lending decisions can aggregate the biases toward the initial choice of the approved applicants because populations who never get loan approval are not contained in the data used to learn the decision policy. Unlike the traditional approach, under our proposed policy, people who are less likely to get approved have a non-zero probability of approval to ensure diversity in the training data.

For the decomposable rewards, we consider

$$q = \frac{1}{n} \sum_{j \in U(\hat{s})} \phi[j]\hat{s}[j] + \epsilon[j]. \tag{17}$$

In this case, $z = [\phi^\intercal, \epsilon^\intercal, \gamma^\intercal] = [\phi^\intercal, \epsilon^\intercal]$ as $\gamma_\xi = 0, \forall \xi$ and n is the number of features. $F_{z_t} = \big[F_{z_t}[1], F_{z_t}[2], \cdots F_{z_t}[2n]\big]^\intercal$ is given by

$$F_{z_t}[k] = \frac{1}{N_t} \sum_{i=1}^{N_t} w_{i,t}[k] \big(R(\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}) - \bar{R}_t\big), \tag{18}$$

$$w_{i,t}[k] = \frac{1}{\pi_{z_t}(\hat{s}_{i,t}, m_{i,t}, a_{i,t})} \frac{\partial \pi_z(\hat{s}_{i,t}, m_{i,t}, a_{i,t})}{\partial z[k]}, \tag{19}$$

$$\bar{R}_t = \frac{1}{t-1} \sum_{\tau=1}^{t-1} \frac{1}{N_\tau} \sum_{i=1}^{N_\tau} R(\hat{s}_{i,\tau}, m_{i,\tau}, a_{i,\tau}, b_{i,\tau}). \tag{20}$$

Here, $\frac{\partial \pi_{z_t}(\hat{s}_{i,t}, m_{i,t}, a_{i,t})}{\partial z[k]} = g(\hat{s}, k)\frac{dL(q)}{dq}$ is the partial derivative of $\pi_z(\hat{s}_{i,t}, m_{i,t}, a_{i,t})$ with respect to the k-th entry of z evaluated at z_t, where g is defined to be

$$g(\hat{s}, k) = \begin{cases} \hat{s}[k]; & k \le n, k \in U(\hat{s}), \\ 1; & k \ge n+1, k \in U(\hat{s}), \\ 0; & \text{otherwise.} \end{cases} \tag{21}$$

For the non-decomposable rewards, we consider

$$q = \frac{1}{n} \sum_{j \in U(\hat{s})} \phi[j]\hat{s}[j] + \epsilon[j] + \sum_{\xi} \gamma_\xi \, \mathbb{1}\{\hat{s} \in \hat{S}_\xi\}, \tag{22}$$

where $\mathbb{1}\{\hat{s} \in \hat{S}_\xi\}$ is an fairness indicator function of $(\hat{s} \in \hat{S}_\xi)$. $\hat{S}_\xi$ refers to feature values that may introduce discrimination in microfinance lending decisions, such as gender, race, ethnicity, etc. In this case, $z = [\phi^\intercal, \epsilon^\intercal, \gamma^\intercal]^\intercal$, $\gamma = [\gamma_1, \gamma_2, \gamma_3, \cdots]^\intercal$, and $F_{z_t} = \big[F_{z_t}[1], F_{z_t}[2], \cdots F_{z_t}[2n], \cdots, F_{z_t}[2n + |\Xi|]\big]^\intercal$, where F_{z_t} is given by

$$F_{z_t}[k] = \mathcal{R}(\{s_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i \in \mathcal{N}_t}) w_t[k], \tag{23}$$

$$w_t[k] = \sum_{i=1}^{N_t} w_{i,t}[k], \tag{24}$$

$$w_{i,t}[k] = \frac{1}{\pi_{z_t}(\hat{s}_{i,t}, m_{i,t}, a_{i,t})} \frac{\partial \pi_z(\hat{s}_{i,t}, m_{i,t}, a_{i,t})}{\partial z[k]}, \tag{25}$$

$$\bar{R}_t = \frac{1}{t-1} \sum_{\tau=1}^{t-1} R(\{s_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}\}_{i \in \mathcal{N}_t}). \tag{26}$$

Here, $\frac{\partial \pi_{z_t}(\hat{s}_{i,t}, m_{i,t}, a_{i,t})}{\partial z[k]} = g(\hat{s}, k)\frac{dL(q)}{dq}$ with g is defined to be

$$g(\hat{s}, k) = \begin{cases} \hat{s}[k]; & k \le n, k \in U(\hat{s}), \\ 1; & n+1 \le k \le 2n, k \in U(\hat{s}), \\ \gamma_{k-2n}; & 2n+1 \le k \le 2n + |\Xi|, \\ 0; & \text{otherwise.} \end{cases} \tag{27}$$

In both cases, the value of $R(\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}) - \bar{R}_t$ and the update size of z_t will become small when $\bar{R}_t$ is sufficiently close to the sample average. The above procedures are summarized in Algorithm 1.

Algorithm 1: Policy Update

Initialized z_1
for each lending period t **do**
  for each application i **do**
    Generate the decision of application i with:

$$a_{i,t} = \begin{cases} 1; & \text{with probability } \pi_{z_t}(\hat{s}_{i,t}, 1), \\ 0; & \text{with probability } \pi_{z_t}(\hat{s}_{i,t}, 0). \end{cases}$$

    Observe outcome $b_{i,t} \in \{0, 1\}$.
    Gain utility $R(\hat{s}_{i,t}, a_{i,t}, b_{i,t})$.
  end for
  Compute F_{z_t} from (18) when choosing general case, or (23) when choosing fairness case.
  Update z_{t+1} based on (13) and (14).
end for

Our proposed algorithm considers the following objectives:

1. Financial inclusion and exploration. The stochastic policy provides approval probability $\mathbb{P}(A|\hat{S})$ rather than a specific lending decision (approve or reject), which ensures sufficient diversity in samples.

2. Account for missing data. The model contains parameter $\epsilon[j]$ to differentiate zero-value data and empty data.
3. Fairness consideration for heterogeneous features. We consider three types of fairness: independence, outcome fairness, and statistical parity which may introduce discrimination against applicants of certain types. We also consider their solutions for the fair allocation of resources for different groups of applicants.
4. Each piece of unbiased information contributes to the policy. Each feature will either positively or negatively contribute to the probability of approval.
5. Convergence under mild assumptions. Specifically, we will show that the average utility approaches the optimum in the long run (see Theorem 4.1 and Corollary 4.2).
6. Parameters learned directly from the algorithm.

4 Optimality and Convergence Analysis

This section provides conditions that ensure the proposed algorithm converges to optimal parameters (Theorem 4.1). Along the way, we explain the ideas behind the updating rules (13) in Lemma 4.3. we also find an appropriate choice for the step size α_t in Corollary 4.2, based on the results of Theorem 4.1.

Convergence condition. Algorithm 1 converges to the optimal parameters when the following conditions are fulfilled:

1. $L(q)$ is a concave function.
2. The set of admissible policy parameters $\mathcal{Z}$ satisfies
$$\mathbb{E}\left[\|z_{t_1} - z_{t_2}\|^2\right] \leq D^2,\ \forall z_{t_1}, z_{t_2} \in \mathcal{Z}. \tag{28}$$
3. The second moment of the stochastic gradient is bounded,
$$\mathbb{E}\left[\|F_{z_t}\|^2 \mid z_t\right] \leq G^2. \tag{29}$$

The convergence conditions are formally stated in the theorem below.

Theorem 4.1. *Assuming conditions 1, 2, and 3 hold, let $C(T)$ be defined by*

$$C(T) = \sum_{t=1}^{T} \alpha_t, \tag{30}$$

and $z^* = (\phi^*, \epsilon^*, \gamma^*)$ be defined in (10)*. Then, Algorithm 1 gives the following performance:*

$$\mathbb{E}\left[\sum_{t=1}^{T} \big(V(z^*) - V(z_t)\big)\right] \leq \frac{1}{2}\left(\frac{1}{\alpha_T} D^2 + G^2 C(T)\right). \tag{31}$$

Theorem 4.1 gives the following relation between step size and convergence speed.

Corollary 4.2. *When step size is chosen to be $\alpha_t = \dfrac{D}{G\sqrt{t}}$, we have*

$$\mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} \big(V(z^*) - V(z_t)\big)\right] \leq \frac{3DG}{2\sqrt{T}}. \tag{32}$$

Theorem 4.1 relies on the property that

$$\mathbb{E}\left[F_{z_t} \mid z_t\right] = \nabla_z V(z_t). \tag{33}$$

Lemma 4.3. *When the total reward can be decomposed as the sum of rewards from individual applicants as in* (4)*, the updating rules* (13) *and* (14) *given* (18) - (21) *satisfy* (33)*.*

Lemma 4.4. *When the total reward cannot be decomposed as the sum of rewards from individual applicants, the updating rules* (13) *and* (14) *given* (23) - (27) *will also satisfy* (33)*.*

Here, from an online learning perspective, one can interpret that the algorithm performs a form of stochastic gradient on reward function V in the presence of missing information where, from a control perspective, $V(z)$ can be interpreted as a Lyapunov function. From Theorem 4.1, optimization problem (10) converges to the optimal parameters as $T \to \infty$ for the concave utility function. When the utility function of interest is not concave, further care might be needed. The Proposition 4.5 gives conditions under which the utility function is concave.

Proposition 4.5. *If the approval probability $L(q)$ is a concave function of q, then the objective function $V(z)$ is concave in z.*

The decision rule (16) in Section 2 is an example of concave function in q over the positive domain. Proposition 4.5 and condition 1 imply the expected rewards derived from concave decision rules such as (16) are also concave. The proofs of the above theorem, corollary, lemma, and proposition are derived in the Appendix C.

5 Experiment and Results

We investigated the empirical behaviors of the proposed methods in a variety of settings. The performance of the proposed algorithm was compared with the perceptron, credit score based method, random forest, support vector machine (SVM), and logistic regression (see Appendix D for their detailed description). The application data was generated from more than 30 different distributions. The distributions were constructed both randomly (see Tables 2 to 4) and from historical loan application data (Dorfleitner and Oswald 2016; Hartley 2010). We considered the utility function in the following form,

$$R(\hat{s}, m, a, b) = \begin{cases} m(r+e); & a=1, b=1, \\ m(-1+e); & a=1, b=0, \\ 0; & a=0. \end{cases} \tag{34}$$

Here, $e \in \mathbb{R}_+$ is the financial inclusion factor to motivate the MFI toward approving more applications[5]. In our study, we set $r = 0.35$ (Kneiding and Rosenberg 2008). The rise time in our simulation is defined as the number of time steps to achieve $(1 \pm 0.01)R(\hat{S}, M, A, B)$. The simulations were performed on a computer with an Intel Core i7-10875H processor with 32 GB of Random Access Memory (RAM). The comparison can be seen in Figure 2.

[5]MFIs often receive subsidies from international development agencies and governments to help offset high risks of lending without collateral.

Robustness against missing data. Figure 2a shows the performance degradation of the proposed and existing algorithms to a varying level of missing feature information. The detailed settings are described in Appendix E. While the performance of all algorithms decreased with the ratio of missing entries, the proposed algorithm degraded more gracefully than others. This is because, unlike the other algorithms, the proposed algorithm is designed to differentiate the empty entry so that the missing information does not contribute to the decision policy (see Section 2), resulting in an approach that is robust against missing information. Moreover, the frequency of missing information makes the purely data-driven methods (*e.g.*, logistic regression, decision tree, random forest) difficult to be applied.

Adaptation to changes. We studied the adaptability of the proposed algorithm to the dynamics in the social and economic conditions by changing the distribution of the dataset in the middle of the simulation. Figure 2b shows the performance of the tested algorithms when the application distribution changes at the 150th lending period, without the knowledge of if and when the distribution has changed (see Appendix D for the detailed settings). As can be seen, the proposed algorithm recovered faster than the other algorithms as it uses immediate feedback from the latest samples to perform quick adaptation. In contrast, the other algorithms, which are primarily designed for offline use, end up using the data that contains the samples from both before and after the changes.

Ability to deal with diverse microfinance distributions. We examined the performance of the compared algorithms using the dataset from 30 different random distributions as well as the distributions inspired from the dataset in kiva.org (see Appendix D). Figures 2c and 2f capture the statistic of the normalized steady-state utilities and the rising time of the algorithms, respectively, where the data were taken from the random distributions and there was 10% missing information. Figures 2d and 2g capture the statistic of the normalized steady-state utilities and the rising time of the algorithms, respectively, where the data were taken from the distributions inspired by the dataset from kiva.org with both 0 and 20% missing information. On average, the proposed algorithm converged to a higher utility than the other algorithms, suggesting that the proposed algorithm can learn a more optimal policy even when there is incomplete information, achieving our first design goal. The proposed algorithm also has a competitive learning speed compared to the other algorithms, achieving our second design goal.

Group dynamics. The group lending setting can be seen in Appendix D. The algorithms take group size as part of the features together with other personal information. The results of group utility and rising time are shown in Figures 2e and 2h. The statistical summary of the final average normalized utilities from the 18 different distributions listed in Table 2 with advanced group lending adaptation, obtained by each algorithm, can be seen in Figure 2e, with their rise time are showed in Figure 2h. Here, we can see that our proposed method provided higher converged utilities with lower rise time at most of the considered distributions. From the statistical summary, on average, our proposed approach converged to the optimal utilities faster with relatively small uncertainty. This result shows that the proposed approach can better seize and learn the influence caused by different group sizes.

Ensuring fairness among different groups Figure 2j shows the box plot for the acceptance rate for the minority group over the course of the simulation. For type 2 fairness, we can see that by setting the target ratio $\Pi(\xi) = 0.4$, the mean of the acceptance rate is above 0.4. For type 3 fairness, the gap in acceptance rate for both groups stayed relatively small throughout. Figure 2j shows the impact on average utility as we change the target acceptance ratio. We can see that enforcing a 50% acceptance rate only cost us less than 10% loss on average utility.

Improved tradeoffs between default risk vs. financial inclusion. There is a tradeoff between default risk vs. financial inclusion because a higher approval rate comes at the expense of higher default risks. Figure 2k shows such tradeoffs for the algorithms tested. The detailed settings are described in Appendix E. The proposed algorithm allows us to systematically tradeoff default risk and financial inclusion through varying the loan subsidy level e. The perceptron, random forest, SVM, and logistic regression algorithms do not have the flexibility to do so because they cannot be optimized for a utility function. The credit score based method is not visible in the current plot range due to large performance degradation in the presence of missing data. The proposed algorithm achieved a reduced default rate for an identical approval rate. The risk-inclusion tradeoffs (solid lines) can be further improved by exploiting group association and liability (dotted lines). This is achieved by having competitive/better steady-state utility and learning speed that are scalable to high dimensional feature space and robust to missing data (Figures 2c and 2f).

Summary. The empirical experiments suggest the proposed algorithm has competitive performance in terms of robustness against missing data, speed of adaptation, and ability to deal with diverse application distributions, thereby achieving improved tradeoffs between default risk vs financial inclusion. The experiment also shows that the proposed algorithm can deal with group dynamics and eliminate fairness issues.

6 Conclusion

In this work, we presented a novel control-theoretic model for microfinance lending strategy. The model solves three main challenges in microfinance: (a) the insufficient past data problem, (b) the missing applicants' information problem, and (c) the group liability structure. Extensive empirical results from numerous synthetic datasets showed several notable performances upon benchmark models, such as robustness against missing data, adaption to changes, group lending scenario, convergence speed, fairness tradeoff, and default risk vs. financial inclusion tradeoff. In addition, we proposed several penalty methods for different fairness scenarios to avoid introducing discrimination to the decisions. We hope our model will be useful for achieving the United Nation's Sustainable Development Goals and could help more people in the under-developed regions have a better life.

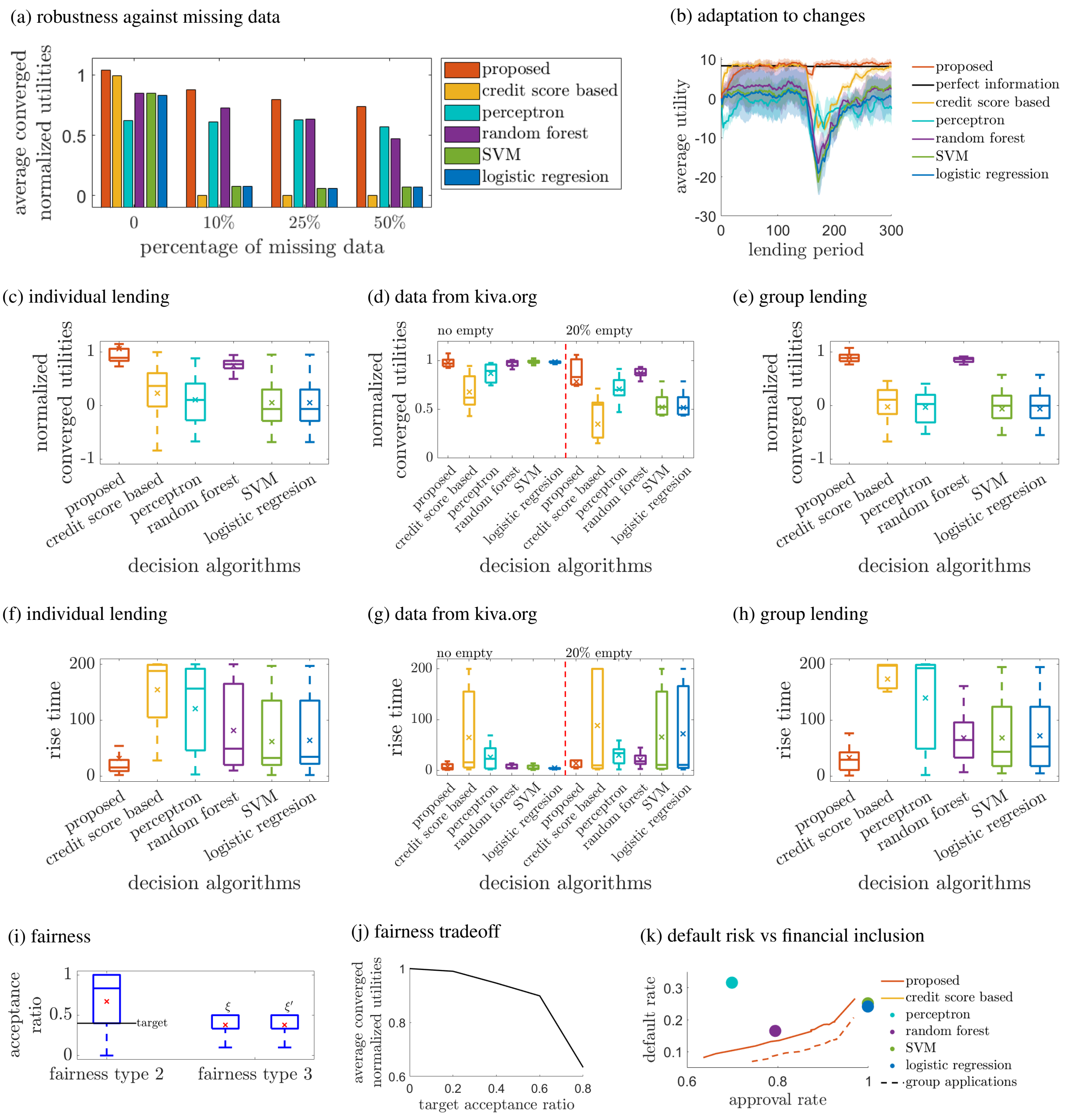


Figure 2: (a) Average converged cumulative normalized utilities of 50 simulations are shown for varying rates of missing data. The utility values are normalized by the maximum utility when the exact return/default probability is known. (b) The average utility when the distribution changes at the 150th lending period. The statistics of the steady state utility for (c) diverse individual applicant distributions, (d) dataset from kiva.org, and (e) group lending case. The statistics of the rise time (learning speed) for (f) diverse applicant distributions, (g) dataset from kiva.org, and (h) group lending case. (i) Acceptance ratio statistics of the discriminative features to show fairness types 2 and 3. (j) The tradeoff resulted from the fairness. (k) Tradeoffs between default probability (risk) vs approval rate (financial inclusion). In subfigures (c) to (i) the mean values are shown by '$\times$', and the first, second, and third quartiles, as well as the maximum and minimum values, are shown as the boxplot. Except for subfigure (a) and the result from the kiva.org dataset, the results were generated for 10% of missing information. Except for (k), we set $e = 0$.

A Related Work

Tools developed for regular finance. The most standard approach to decide regular finance is based on credit scores, which inform the likelihood of each application to default (Klaff 2004; Puro et al. 2010). Other approaches consider the loan approval process as a binary classification problem to be solved using machine learning methods such as discriminant analysis (Baesens et al. 2003), logistic regression (Ala'raj and Abbod 2016; Vaidya 2017), and neural networks (Abdou, Pointon, and El-Masry 2008; Chen, Zhang, and Ng 2018; Condori-Alejo, Aceituno-Rojo, and Alzamora 2021). Multi-layers perceptron neural networks are widely used in automatic credit scoring systems with high accuracy and efficiency (Zhao et al. 2015; Correa, Gonzalez, and Ladino 2011). To achieve a higher prediction accuracy, some studies utilize the random forest approach with feature selection and grid search to reduce the influence of irrelevant and redundant features (Wang et al. 2012; Van Sang, Nam, and Nhan 2016). Other studies adopt the SVM (support vector machine) algorithm to also improve the prediction accuracy with fewer features (Huang, Chen, and Wang 2007; Chen and Li 2010). In a different direction, some studies predict the default probability of the applicants utilizing logistic regression and its extensions (Bolton et al. 2010; Sohn, Kim, and Yoon 2016). In addition, works such as (Ampountolas et al. 2021) examine and compare various machine learning algorithms to classify borrowers into various credit categories. However, as mentioned, these approaches mostly focused on offline learning, assuming that accurate homogeneous data is abundantly available. These assumptions may not hold in many under-explored (developing) regions, such as rural Africa, where most of the clients' data is missing. Furthermore, collecting such information might be expensive and the data might be unreliable because of the lack of proper state mechanisms.

Fairness in machine learning. Moreover, machine learning approaches may also introduce fairness issues as it is known to be a controversial topic in the field. Especially for the black-box decision-making tool as it will introduce several explicit and implicit biases to the results (D'Amour et al. 2020; Corbett-Davies and Goel 2018; Agarwal 2021; Bantilan 2018; Burrell 2016). A few recent works suggested that the decisions made by these black-box processes may have hidden biases and discrimination against under-served populations (Hall et al. 2021; Chen et al. 2019). Existing approaches to deal with these biases include modification in the data generation process to ensure the data sets have sufficient diversity (Barbierato et al. 2022), as well as pre-, mid-, and post-processing. For example, (Zemel et al. 2013) introduced a pre-processing approach called LFR (Learned Fair Representations), a discriminative clustering model in which the initial data point is mapped to the distribution in a new input space to conceal any information regarding the data point being a member of a protected subgroup, preserving individual fairness. In another example, (Lee et al. 2014) proposed the Fairness-Aware BPRMF method as a mid-prepossessing approach by pairing the Bayesian Personalized Ranking Model (BPR) with a combination of matrix factorization (MF). (Kim, Ghorbani, and Zou 2019) proposed a post-processing algorithm, MULTIACCURACY-BOOST, that contains an auditor algorithm that iteratively makes mistakes on every sub-population in the black box's classified data until the multi-accuracy constraints of equality are satisfied.

Optimization methods. To find the optimal decision policies, the learning algorithms employ existing optimization techniques in stochastic gradient descent (SGD) (Robbins and Monro 1951), reinforcement learning, and optimal control. For instance, bandit algorithms such as AdaGrad (Duchi, Hazan, and Singer 2011), AdaDelta (Zeiler 2012), RMSProp (Mukkamala and Hein 2017), and Adam (Kingma and Ba 2014) are commonly employed. While there is no guarantee for the first-order methods to converge to the optimal solution in general, they will converge to the global optimum solution when the objective function is convex or strongly convex (Zinkevich 2003; Hazan, Agarwal, and Kale 2007).

Handling missing data The learning frameworks need to also handle the missing data problem separately. As missing data is a common real-world problem, there is a rich line of literature dealing with it. The classical methods such as mean imputation (Little and Rubin 2002), expectation maximization (Dempster, Laird, and Rubin 1977; Ghahramani and Jordan 1993; Honaker, King, and Blackwell 2011), least squares (Van Buuren and Groothuis-Oudshoorn 2011; Bø, Dysvik, and Jonassen 2004), K-nearest neighbors (Troyanskaya et al. 2001), and regression tree (Burgette and Reiter 2010), are commonly employed. Recently, an optimization-based approach (Bertsimas, Pawlowski, and Zhuo 2017) has also been considered.

Empirical investigation into microfinance. Extensive efforts have also been devoted to understanding the economics of microfinance and its sustainability (Armendáriz and Morduch 2010; CERISE 2019; Ayayi and Sene 2010; Cull, Demirgüç-Kunt, and Morduch 2018; Van Rooyen, Stewart, and De Wet 2012; Duvendack et al. 2011). The loan default probability has been determined as one key challenge for microfinance's sustainability. Thus, numerous studies have been carried out to identify the contributing factors to microfinance default behaviors (Kamanza 2014; Muthoni 2016; Asongo and Idama 2014; Boadi et al. 2016; Dorfleitner and Oswald 2016). Some features, such as the borrower's gender, education level, family size, residential distance to the institution, lending method, activities financed by the loan, total loan received, and loan monitoring method, among others, have been observed to affect the repayment performance significantly (Field and Pande 2008; Mpogole et al. 2012; Jote 2018; Nawai and Shariff 2010).

Group lending in microfinance. The studies have also identified that group lending is an essential cornerstone of microfinance. Here, loans are made to small groups or cooperatives that made the members share the liability jointly (Schurmann and Johnston 2009; Besley and Coate 1995; Ghatak 1999). This joint-liability model uses social, rather than material, collateral, leveraging peer pressure and community information to overcome asymmetric information in microfinance, leading to better repayment behavior (Mahmud 2020). In this model, when one group member defaults, other members jointly bear the cost. Since members of the group are supposed to know each other, this liability structure will help to overcome the information asymmetrical, inherent in lending to poor borrowers, making group-based lending efficient and effective with low transaction costs for the provider (Armendáriz de Aghion and Morduch 2000; Postelnicu, Hermes, and Szafarz 2014). A study carried out in Pakistan concluded that with group lending, borrowers are about 60% times as likely to miss a payment in any given month under joint liability relative to individual liability (Mahmud 2020). In Africa, there has been also an increasing interest to gear group lending toward traditional group savings structures such as "tontines" in Senegal, "esusu" in Nigeria, "ekub" in Sudan, Eritrea, and Cameroon, or "jangi" in Cameroon (Johnson1, Depesquidoux, and Verges 2021). In these Rotating Savings and Credit Associations (ROSCAs), members meet regularly and pay a predetermined sum of money at each meeting (Kimuyu 1999). The sum of the payments is then given to a group member (usually the host of the meeting), determined via a lottery in the previous meeting, to pay out the loan of this member (Owen 2006). Among these ROSCAs, the women's association, despite its informal nature, has been one of the most resilient communities where they have survived in many areas where formal microfinancing communities have failed. Because of that, microfinancing through women's associations has recently attracted a lot of interest (Perry 2002; Mayoux 2000; Abdallah Ali, Mughal, and Chhorn 2021).

B Notation

We use capital letters for random variables, *e.g.*, A, and lowercase letters for their specific realization, *e.g.*, a. A square bracket is used to represent the entries of a vector, *e.g.*, $s = [s[1], s[2], \cdots, s[n]]^\intercal$, and a regular bracket is used for the input of an function, *e.g.*, $f(x)$. We use $\mathbb{P}(E)$ to denote the probability of an event E or the density function of a random variable E. Lastly, we use $\mathbb{Z}$, $\mathbb{Z}_+$, $\mathbb{R}$, $\mathbb{R}_+$ to denote the sets of integers, non-negative integers, real numbers, and non-negative real numbers.

C Proofs

Proof of Proposition 4.5

For the decomposable case, we have

$$
\begin{aligned}
V(z) &= \mathbb{E}\left[R(\hat{S}, M, A, B)\right] && (35)\\
&= \mathbb{E}\left[\sum_{a=0,1} \mathbb{E}[R(\hat{S}, M, A, B) \mid A = a]\pi_z(\hat{S}, M, a)\right] && (36)\\
&= \mathbb{E}\left[\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 1]\pi_z(\hat{S}, M, 1)\right] && (37)\\
&= \mathbb{E}\left[\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 1]L(q)\right], && (38)
\end{aligned}
$$

where (37) is because $\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 0] = 0$. Since $\mathbb{E}[R(\hat{S}, M, A, B \mid A = 1]$ is independent from z, we can see from (38) that $V(z)$ is a linear combination of $L(q)$ and from (17), q is a linear function of z. Therefore, if q is a concave function of z, then $L(q)$ and $V(z)$ are concave functions of z. □

In the non-decomposable case, we have

$$
\begin{aligned}
V(z) &= \mathbb{E}\left[R(\hat{S}, M, A, B)\right] && (39)\\
&= \mathbb{E}\left[\sum_{a=0,1} \mathbb{E}[R(\hat{S}, M, A, B) \mid A = a]\pi_z(\hat{S}, M, a)\right] && (40)\\
&= \mathbb{E}\left[\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 1]\pi_z(\hat{S}, M, 1) + \mathbb{E}[R(\hat{S}, M, A, B) \mid A = 0](1 - \pi_z(\hat{S}, M, 1))\right] && (41)\\
&= \mathbb{E}\left[\left(\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 1] - \mathbb{E}[R(\hat{S}, M, A, B) \mid A = 0]\right) L(q) + \mathbb{E}[R(\hat{S}, M, A, B) \mid A = 0]\right]. && (42)
\end{aligned}
$$

Since $\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 0]$ and $\mathbb{E}[R(\hat{S}, M, A, B) \mid A = 1]$ are independent from z, we can see that $V(z)$ is a linear combination of $L(q)$. Therefore, $V(z)$ is concave with respect to z when $L(q)$ is concave. □

Proof of Lemma 4.3. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ be the index of the applications in one lending period. Then, let $\hat{\mathbf{s}} = \{\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_N\}$, $\mathbf{m} = \{m_1, m_2, \ldots, m_N\}$, $\mathbf{a} = \{a_1, a_2, \ldots, a_N\}$, and $\mathbf{b} = \{b_1, b_2, \ldots, b_N\}$ be the vectors containing the accessible information, group size, lending decision, and outcomes of all applicants, respectively. For all $k \in \{1, \ldots, 2N\}$, we have

$$
\mathbb{E}\Big[F_z[k] \mid z\Big] = \mathbb{E}\left[\left(\mathcal{R}(\hat{\mathbf{s}}, \mathbf{m}, \mathbf{a}, \mathbf{b}) - \bar{R}\right) \sum_{i-1}^{N} \frac{1}{\pi_z(\hat{s}_i, m_i, a_i)} \frac{\partial \pi_z(\hat{s}_i, m_i, a_i)}{\partial z[k]} \mid z\right] \tag{43}
$$

$$= \mathbb{E}\left[\left(\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R}\right)\left(\frac{\frac{\partial}{\partial z[k]}\pi_z\left(\hat{S},M,A\right)}{\pi_z\left(\hat{S},M,A\right)}\right)\right] \tag{44}$$

$$= \mathbb{E}\Bigg[\sum_{\mathbf{a}}\left(\left(\frac{\pi_z\left(\hat{S},m,a\right)}{\pi_z\left(\hat{S},m,a\right)}\right)\mathbb{E}\Big[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R} \mid A, M, \hat{S}\Big]\left(\frac{\partial}{\partial z[k]}\pi_z\left(\hat{S},M,A\right)\right)\right)\Bigg] \tag{45}$$

$$= \mathbb{E}\Bigg[\mathbb{E}\Bigg[\sum_{\mathbf{a}}\left(\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R}\right)\frac{\partial}{\partial z[k]}\pi_z\left(A,M,\hat{S}\right) \mid A, M, \hat{S}\Bigg]\Bigg] \tag{46}$$

$$= \frac{\partial}{\partial z[k]}\mathbb{E}\Bigg[\mathbb{E}\Bigg[\sum_{\mathbf{a}}\left(\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R}\right)\pi_z\left(A,M,\hat{S}\right) \mid A, M, \hat{S}\Bigg]\Bigg] \tag{47}$$

$$= \frac{\partial}{\partial z[k]}\mathbb{E}\left[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b})\right] \tag{48}$$

$$= \frac{\partial V(z)}{\partial z[k]}. \tag{49}$$

Equality (45) is by law of total probability, (46) is because of the fact that $\bar{R}$ is independent of z while $\sum_{A\in\{0,1\}}\frac{\partial}{\partial z[k]}\pi_z(\hat{S},A) = 0$, and (47) is by linearity of expectation. Here, the expectations in (44) and (48) are taken over $\mathbb{P}(S,\hat{S},A,B)$. The first expectations in (45), (46), and (47) are taken over $\mathbb{P}(S,\hat{S})$ while the second expectations are taken over $\mathbb{P}(B \mid A,\hat{S})$. □

Proof of Lemma 4.4. Using the same notation as in the Proof of Lemma 4.3, we have,

$$\mathbb{E}\left[F_z[k] \mid z\right] = \mathbb{E}\left[\left(\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R}\right)\sum_{i=1}^{N}\frac{1}{\pi_z(s_i,m_i,a_i)}\frac{\partial\pi_z(s_i,m_i,a_i)}{\partial z[k]}\right] \tag{50}$$

$$= \mathbb{E}\left[\sum_{\mathbf{a}}\mathbb{E}\Big[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R} \mid \hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}\Big]\sum_{i=1}^{N}\frac{\pi_z(s_i,m_i,a_i)}{\pi_z(s_i,m_i,a_i)}\frac{\partial\pi_z(s_i,m_i,a_i)}{\partial z[k]}\right] \tag{51}$$

$$= \mathbb{E}\left[\sum_{\mathbf{a}}\mathbb{E}\Big[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) - \bar{R} \mid \hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}\Big]\sum_{i=1}^{N}\frac{\partial\pi_z(s_i,m_i,a_i)}{\partial z[k]}\right] \tag{52}$$

$$= \mathbb{E}\left[\sum_{\mathbf{a}}\mathbb{E}\Big[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) \mid \hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}\Big]\sum_{i=1}^{N}\frac{\partial\pi_z(s_i,m_i,a_i)}{\partial z[k]}\right] \tag{53}$$

$$= \frac{\partial}{\partial z[k]}\mathbb{E}\left[\sum_{\mathbf{a}}\mathbb{E}\Big[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}) \mid \hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b}\Big]\sum_{i=1}^{N}\pi_z(s_i,m_i,a_i)\right] \tag{54}$$

$$= \frac{\partial}{\partial z[k]}\mathbb{E}\Big[\mathcal{R}(\hat{\mathbf{s}},\mathbf{m},\mathbf{a},\mathbf{b})\Big] \tag{55}$$

$$= \frac{\partial V(z)}{\partial z[k]}. \tag{56}$$

Here, (50) is the expectation taken over $\hat{\mathbf{s}},\mathbf{m},\mathbf{a},$ and $\mathbf{b}$; the first expectation in (51) is taken over $\hat{\mathbf{s}}$ and $\mathbf{m}$ while the second expectation is taken over $\mathbf{b}$; (55) holds because $\sum_{a_i\in\{0,1\}}\pi_z(s_i,m_i,a_i) = 1$; (56) is due to the definition of $V(z)$. □

Gradient inequality lemma. To bound the regret of a telescoping sum, we present the following:

Lemma C.1. *The following condition holds:*

$$\mathbb{E}\Big[2\nabla_z V(z_t)\cdot(z_t - z^*) \mid z_t\Big] \le \frac{1}{\alpha_t}\left(\mathbb{E}\Big[\|z_t - z^*\|^2 - \|z_{t+1} - z^*\|^2 \mid z_t\Big]\right) + \alpha_t G^2, \tag{57}$$

where V is defined in Section 2.

Proof. Let us consider $\mathbb{E}\Big[\|z_{t+1}-z^*\|^2 \mid z_t\Big]$, where

$$\mathbb{E}\Big[\|z_{t+1}-z^*\|^2 \mid z_t\Big] = \mathbb{E}\Big[\|\operatorname*{proj}_Z(z_t+\alpha_t F_{z_t})-z^*\|^2 \mid z_t\Big] \tag{58}$$

$$\leq \mathbb{E}\Big[\|z_t+\alpha_t F_{z_t}-z^*\|^2 \mid z_t\Big] \tag{59}$$

$$\leq \mathbb{E}\Big[\|z_t-z^*\|^2+\alpha_t^2\|F_{z_t}\|^2-2\alpha_t F_{z_t}\cdot(z^*-z_t) \mid z_t\Big] \tag{60}$$

$$= \mathbb{E}\Big[\|z_t-z^*\|^2+\alpha_t^2\|F_{z_t}\|^2 \mid z_t\Big]-\mathbb{E}\Big[2\alpha_t F_{z_t}\cdot(z^*-z_t) \mid z_t\Big] \tag{61}$$

$$= \|z_t-z^*\|^2+\alpha_t^2\mathbb{E}\Big[\|F_{z_t}\|^2 \mid z_t\Big]-2\alpha_t\mathbb{E}\Big[F_{z_t}\cdot(z^*-z_t) \mid z_t\Big] \tag{62}$$

$$= \|z_t-z^*\|^2+\alpha_t^2\mathbb{E}\Big[\|F_{z_t}\|^2 \mid z_t\Big]-2\alpha_t\nabla_{z_t}V(z_t)\cdot(z^*-z_t), \tag{63}$$

therefore,

$$2\nabla_{z_t}V(z_t)\cdot(z^*-z_t) \leq \frac{-\mathbb{E}\Big[\|z_{t+1}-z^*\|^2 \mid z_t\Big]+\|z_t-z^*\|^2}{\alpha_t}+\frac{\alpha_t^2\mathbb{E}\Big[\|F_{z_t}\|^2 \mid z_t\Big]}{\alpha_t} \tag{64}$$

$$\leq \frac{-\mathbb{E}\Big[\|z_{t+1}-z^*\|^2 \mid z_t\Big]+\|z_t-z^*\|^2}{\alpha_t}+\alpha_t G^2, \tag{65}$$

thus,

$$\mathbb{E}\Big[2\nabla_{z_t}V(z_t)\cdot(z^*-z_t) \mid z_t\Big] \leq \frac{-\mathbb{E}\Big[\|z_{t+1}-z^*\|^2 \mid z_t\Big]+\|z_t-z^*\|^2}{\alpha_t}+\alpha_t G^2. \tag{66}$$

Here, (63) is due to Lemma 4.3 and (65) is due to the assumption $\mathbb{E}\left[\|F_{z_t}\|^2 \mid z_t\right] \leq G^2$. Thus, we can then recover (57). □

Proof of Theorem 4.1. Given Lemmas C.1 and 4.3, we can proof Theorem 4.1:

$$\mathbb{E}\left[2\sum_{t=1}^{T}\big(V(z^*)-V(z_t)\big)\right] \leq \mathbb{E}\left[2\sum_{t=1}^{T}\big(\nabla_z V(z_t)\cdot(z^*-z_t)\big)\right] \tag{67}$$

$$\leq \mathbb{E}\left[2\sum_{t=1}^{T}\mathbb{E}\Big[\nabla_z V(z_t)\cdot(z^*-z_t) \mid z_t\Big]\right] \tag{68}$$

$$\leq \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathbb{E}\left[\|z_{t+1}-z^*\|^2+\|z_t-z^*\|^2 \mid z_t\right]}{\alpha_t}+G^2\sum_{t=1}^{T}\alpha_t\right] \tag{69}$$

$$\leq \sum_{t=1}^{T}\mathbb{E}\left[\|z_t-z^*\|^2\right]\left(\frac{1}{\alpha_t}-\frac{1}{\alpha_{t-1}}\right)+G^2\sum_{t=1}^{T}\alpha_t \tag{70}$$

$$\leq D^2\sum_{t=1}^{T}\left(\frac{1}{\alpha_t}-\frac{1}{\alpha_{t-1}}\right)+G^2\sum_{t=1}^{T}\alpha_t \tag{71}$$

$$\leq \frac{1}{\alpha_T}D^2+G^2C(T). \tag{72}$$

Here, (67) is due to the concavity of $V(z_t)$; (68) is due to the law of total expectation; (69) is due to Lemma C.1; (70) is due to the law of total expectation; (71) is due to assumption (28); and (72) is due to assumption (30). □

Proof of Corollary 4.2. We have

$$\mathbb{E}\left[\sum_{t=1}^{T}\left(V(z^*)-V(z_t)\right)\right] \leq \frac{1}{2}\left(D^2\sum_{t=1}^{T}\left(\frac{1}{\alpha_t}-\frac{1}{\alpha_{t-1}}\right)+G^2\sum_{t=1}^{T}\alpha_t\right) \tag{73}$$

$$\leq \frac{1}{2}\left(D^2\frac{G\sqrt{T}}{D}+G^2\frac{D}{G}2\sqrt{T}\right) \tag{74}$$

$$= \frac{1}{2}\left(DG\sqrt{T} + 2GD\sqrt{T}\right) \tag{75}$$

$$= \frac{3}{2}DG\sqrt{T}, \tag{76}$$

where (73) is due to (71); (74) is due to $\sum_{t=1}^{T} \frac{1}{\sqrt{t}} \leq 2\sqrt{T}$. We recover (32) by dividing (76) by T. □

D Experiment and Simulation Setting

Generating Application Data Pools

To test our algorithm, we generated numerous synthetic and artificial data pools where each data pool contains 10^6 data samples.

Realistic synthetic dataset from Kiva dataset. We augmented a real microfinance loan dataset based on applicants' features and information from the Kiva platform (Hartley 2010)[6]. The Kiva dataset contains the features and returns/default information of $N = 3,181$ approved loan applications in 2011. There are 41 features in the raw Kiva dataset, and we categorized the application features into several categories shown in Table 1. In our study, we only considered the features with the "*used*" label. After filtering, there are 15 features that we used in our study. We used the "posted date" and "funded date" information to obtain the duration until the loan is funded from the date when the application is posted on the platform. After filtering the features, we first converted all the descriptive features (*e.g.*, country name, activity sector for requested loan) to numerical values based on the defaulted rate of applications. To ease the optimization process, we map the numerical values with a maximum possible value above 4 to the $[0, 4]$ range, utilizing the following process,

$$s_{\text{scaled}}[k] = \frac{4 \cdot s_{\text{kiva}}[k]}{\max(s_{\text{kiva}}[k])}. \tag{77}$$

Here, $\max(s[k])$ refers to the maximum value of the k feature. We then generated synthetic data from the Kiva dataset using the following two different methods.

First, we augmented the pre-processed data by fitting it into the following parametric model,

$$\mathbb{P}_{\rho,\nu}(B \mid \hat{S}) = \frac{1}{1 + e^{-x}}, \tag{78}$$

$$\tag{79}$$

where $x \in \mathbb{R}$ is a scalar value computed as

$$x = \sum_{k} \rho[k]\hat{S}[k] + \nu[k]. \tag{80}$$

Here, $\{\rho, \nu\} \in \{\mathbb{R}^n, \mathbb{R}^n\}$ are the parameters that govern the shape of the model. We generated the synthetics realistic dataset consisting of 10^6 data samples by assigning random feature values drawn from the pre-processed dataset. Then, we drew each application outcome independently from the parametric model, *i.e.*,

$$b_{i,t} \overset{\text{i.i.d.}}{\sim} \mathbb{P}_{\rho^*,\nu^*}(B \mid \hat{S}). \tag{81}$$

Here, we considered a default probability to be around 25%, the default rate of microfinance observed in Africa (Pollio and Obuobie 2010).

Second, we augmented the pre-processed data utilizing a bootstrapping method. Here, we first differentiate the data into two categories, defaulted and paid. We then generated pools of datasets consisting of 10^6 applications each with 10%, 20%, 30%, 40%, and 50% default rates by randomly drawing applications from the pre-processed data. To achieve the desired default rates, we drew the defaulted and paid data separately. This method keeps the distributions of the original Kiva dataset, but cannot produce new samples that are not contained in the original dataset. In contrast, the previous method uses the original distribution of the features but can create new samples by randomly sampling B using the learned return probability from the original dataset. We use data generated from both methods to understand how the algorithm behaves in different scenarios.

Artificial distributions for individual applications. We generated artificial data pools for individual applications based on 30 different distribution types. We considered the feature vector S of 100-features information from each loan application. Each feature contains a non-negative number that represents the applicant's information. Those features could go from personal information, including age range, income level, education level, language skill, etc., to household information such as household type, number of bedrooms, internet accessibility, etc. For distribution types 1 through 26, we bounded the values of the feature

[6]The original kiva dataset can be accessed from https://stat.duke.edu/datasets/kiva-loans

Feature Number	Feature Name	Feature Description	Data Preprocessing Decision
1	id	loan ID	unique for each applicant
2	description.languages	language of loan description	used
3	funded_amount	amount of loan has been collected by lenders	not available in the beginning
4	paid_amount	amount of the loan which has been paid off	not available in the beginning
5	activity	activity for requested loan	used
6	Sector	sector for requested loan	used
7	location.country_code	country code	used
8	location.country	country name	used
9	location.town	town name	used
10	location.geo.level	latitude and longitude indicator for country or town	used
11	partner_id	partner ID	irrelevant
12	borrowers.first_name	first name of borrower	unique for each applicant
13	borrowers.last_name	last name of borrower	unique for each applicant
14	borrowers.gender	gender of borrower	used
15	borrowers.pictured	borrowers' picture availability	irrelevant
16	terms.disbursal_amount	distributed amount in the local currency	conflict with our assumption
17	terms.disbursal_currency	distributed currency	used
18	terms.disbursal_date	distributed date	conflict with our assumption
19	paid_date	Fully paid date	not available in the beginning
20	defaulted_date	defaulted date	not available in the beginning
21	terms.loan_amount	the amount of money distributed	conflict with our assumption
22	terms.loss_liability.nonpayment	who is liable for non repayment	used
23	terms.loss_liability.currency_exchange	who is liable for currency exchange loss	used
24	posted_date	loan posted date on Kiva	to calculate the duration: used
25	funded_date	fully funded date	
26	journal_total.entries	number of updates by borrower	not available in the beginning
27	terms.local_payments.due_date	payment is due date to the field partner	conflict with our assumption
28	terms.local_payments.amount	amount due to the field partner	conflict with our assumption
29	terms.scheduled_payments.due_date	scheduled payment due date	conflict with our assumption
30	terms.scheduled_payments.amount	scheduled payment due amount	conflict with our assumption
31	delinquent	whether has become delinquent	not available in the beginning
32	video.youtube_id	youtube id if provide a video	irrelevant
33	basket_amount	amount of loan saved but not confirmed	irrelevant
34	amount	payment amount in US dollars	conflict with our assumption
35	payment_id	payment ID	irrelevant
36	local_amount	payment amount in local currency	conflict with our assumption
37	processed_date	processed date	irrelevant
38	rounded_local_amount	rounded local payment amount	conflict with our assumption
39	settlement_date	payment settlement date	conflict with our assumption
40	lat	latitude of loan location	used
41	lon	longitude of loan location	used
42	status	paid or defaulted status	label

Table 1: Description of the features and data pre-processing decision of the Kiva dataset.

information in the range of $[0, 4]$, while distribution types 27 through 30 have unbounded values of feature information. The feature information for distribution types 1 through 18 were generated based on

$$\mathbb{P}(S)_l = a_S + \frac{2 - 2a_S b_S}{b_S(b_S - 1)} (\text{bin}_l - 1), \tag{82}$$

which is parameterized by constants a_S and b_S. Here, bin_l is the l-th bin of the feature domain, and $\mathbb{P}(S)_l$ is the feature distribution in the l-th bin. In our simulation we used $a_S = \{0,\ 0.005,\ 0.01\}$ and $b_S = 100$. Specific distribution functions to generate feature information for distribution types 19 through 30 can be seen in Tables 3 and 4. We defined the return probability of individual lending, $\mathbb{P}(B \mid \hat{S}, M = 1)$, based on $\mathbb{P}(\hat{S} \mid S) = 1$, where we considered three forms of $\mathbb{P}(B \mid \hat{S}, M)$:

$$\mathbb{P}\left(B = 1 \mid \hat{S}, M\right) = c_{B1}(q_B) + c_{B2}, \tag{83}$$

$$\mathbb{P}\left(B = 1 \mid \hat{S}, M\right) = c_{B1}(q_B)^2 + c_{B2}(q_B) + c_{B3}, \tag{84}$$

$$\mathbb{P}\left(B = 1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1 + \exp(q_B)}. \tag{85}$$

Detailed values for constants $c_{B1},\ c_{B2},$ and c_{B3} as well as the specific form of parameter q_B can be seen in Tables 2 to 4.

Artificial distributions with group applications and liability. We considered two cases of group lending strategy: the basic case with empty heterogeneous information $\pi(S = \emptyset, M, A)$ and the advanced case $\pi(S \neq \emptyset, M, A)$. For the basic case, the data pool only contains group size information. Each application i comes with group size m. In the empirical study, we assume that the group size is uniformly distributed among $\mathcal{G} = \{1, 2, \cdots, 100\}$,

$$\mathbb{P}(M = m) = \begin{cases} 1/|\mathcal{G}| & \text{if } m \in \mathcal{G} \\ 0 & \text{otherwise.} \end{cases} \tag{86}$$

The liability for the loan repayment is imposed on the group by obligating the members of the group to cover the other members who cannot return the loan and its interest, $1 + r$. At the end of the lending period, each applicant j in the group i holds $\theta_{i,j}$ unit of money, which is the principal plus gains or loss. We assume that $\theta_{i,j}$ is an *i.i.d.* Gaussian random variable, *i.e.*,

$$\theta_{i,j} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mu(\hat{s}_i), \sigma^2). \tag{87}$$

where $\mathcal{N}(\mu, \sigma^2)$ refers to a Gaussian distribution with mean μ and standard deviation σ. Thus, at the end of the lending period, the group i holds $\sum_{j=1}^{m_i} \max\{\theta_{i,j}, 0\}$ units of money in total. Here, we assumed that the members with $\theta_{i,j} < 0$, who hold debts from elsewhere, do not have the capability to return any money but will not transfer the other debts to the group. Because group i must return $m_i \cdot (1 + r)$, the group will default if

$$\sum_{j=1}^{m_i} \max(\theta_{i,j}, 0) < m_i \cdot (1 + r). \tag{88}$$

Accordingly, the return probability for group application with m_i members is given by

$$\mathbb{P}(B = 1 \mid \hat{S} = \hat{s}_i, M = m_i) = \mathbb{P}\left(\sum_{j=1}^{m_i} \max(\theta_{i,j}, 0) \geq m_i \cdot (1 + r)\right). \tag{89}$$

For basic case data pool, we choose $\mu = 1.42$ and $\sigma = 0.5$. For advanced case data pool, we considered the repayment probability types 1 to 18 in Table 2, where we modeled $\mu(\hat{s}) = \Delta + \mathbb{P}(B = 1 \mid \hat{S} = \hat{s}, M = m)$ and $\sigma = 0.5$ given $\Delta = 0.5 + r$ for $r = 0.35$, that helped to center μ around a reasonable repaid amount.

Simulation Setting

We performed simulations for lending periods of $t = 1$ to $t = 500$. At each lending period, we choose $N_t = 10$ applicants, taken from the pool of applicants generated using methods described in Appendix D. Here, we have the observable information $\hat{s}_{i,t}$.

Algorithms to compare. We compared our proposed algorithm against the following existing algorithms:

- **Perfect Repayment Information.** The best scenario to decide on loan approval is when perfect knowledge of repayment probability is available. Here, for the perfect decision making, (34) can be rewritten as

$$R(\hat{S}, M, A, B) = \begin{cases} (r + e) \cdot m; & \mathbb{P}(B = 1 \mid A = 1, S, M = m), \\ (-1 + e) \cdot m; & \mathbb{P}(B = 0 \mid A = 1, S, M = m), \\ 0; & A = 0, B = 0, \end{cases} \tag{90}$$

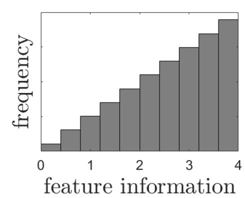
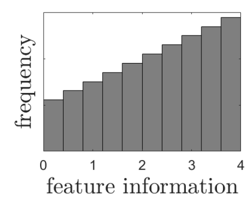
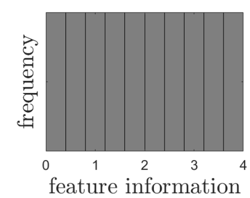
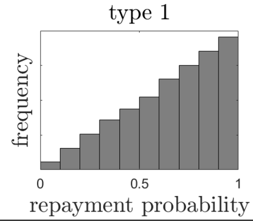
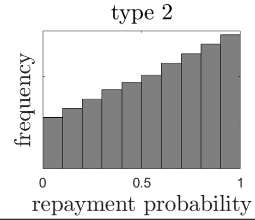
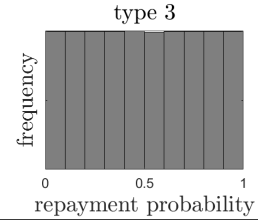
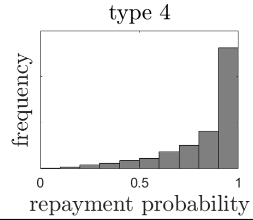
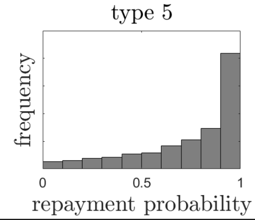
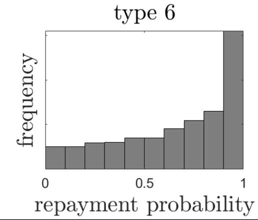
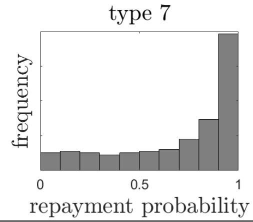
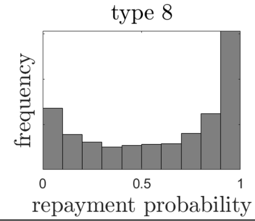
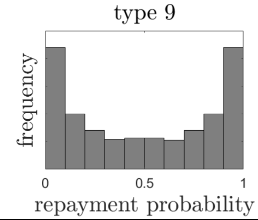
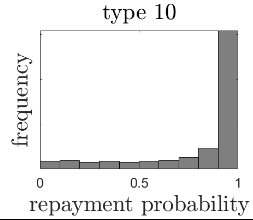
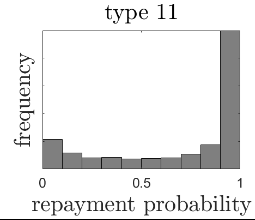
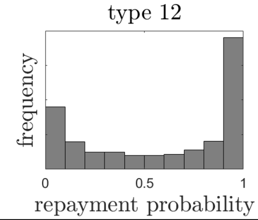
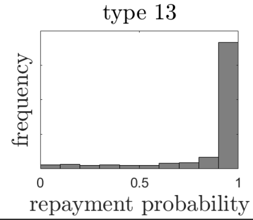
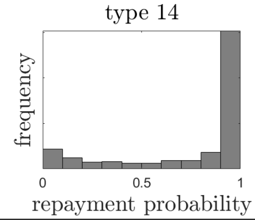
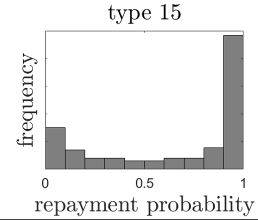
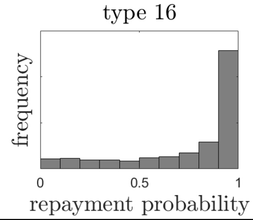
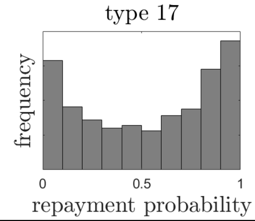
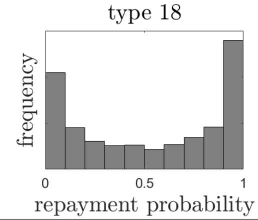
features distribution, $\mathbb{P}(S)$ / repayment probability, $\mathbb{P}\left(B=1 \mid \hat{S}, M\right)$			
			
$q_B = \frac{1}{n}\sum_{j=1}^{n} s[j]$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{1}{4}q_B$	type 1 	type 2 	type 3 
$q_B = \frac{1}{n}\sum_{j=1}^{n} s[j]$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = -\frac{1}{16}q_B^2 + \frac{1}{2}q_B$	type 4 	type 5 	type 6 
$q_B = \frac{1}{n}\sum_{j=1}^{n} 2s[j] - 4$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$	type 7 	type 8 	type 9 
$q_B = \frac{1}{n}\sum_{j=1}^{n} 2.5s[j] - 4$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$	type 10 	type 11 	type 12 
$q_B = \frac{1}{n}\sum_{j=1}^{n} 3s[j] - 4$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$	type 13 	type 14 	type 15 
$\{\mathscr{W}[1], ..., \mathscr{W}[n]\} \sim \mathcal{N}(2, 4^2)$ $q_B = \frac{1}{n}\sum_{j=1}^{n} \mathscr{W}[j]s[j] - 4$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$	type 16 	type 17 	type 18 

Table 2: List of the repayment probability distributions considered in our study.

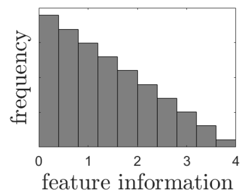
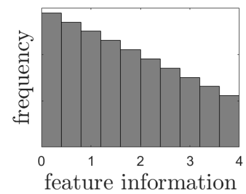
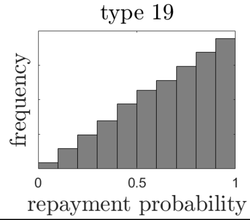
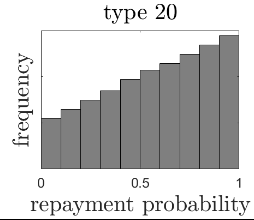
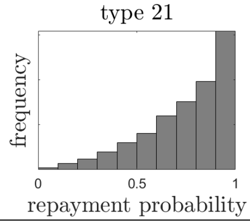
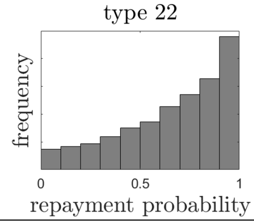
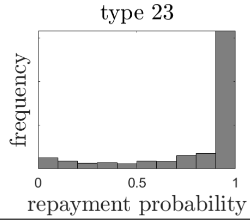
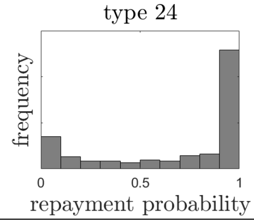
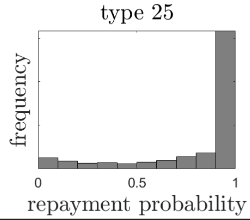
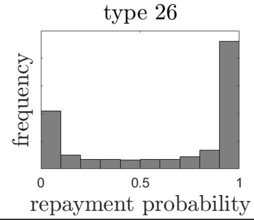
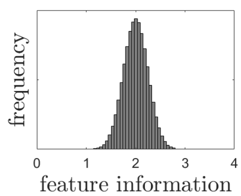
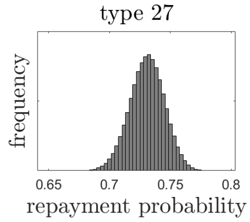
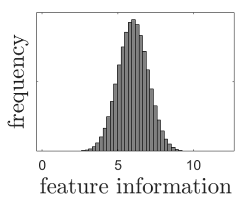
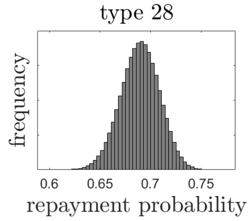
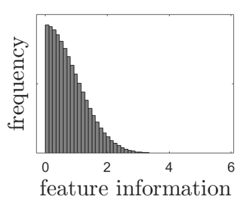
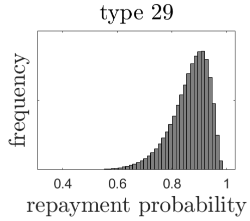
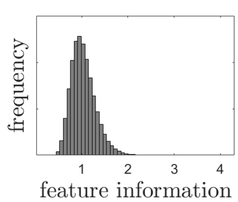
repayment probability, $\mathbb{P}\left(B=1 \mid \hat{S}, M\right)$ \ features distribution, $\mathbb{P}(S)$	 $\mathbb{P}(S)_l = \frac{2}{99} - \frac{bin_l}{4950}$	 $\mathbb{P}(S)_l = \frac{1}{99} - \frac{bin_l}{9900}$
$q_B = \frac{1}{n}\sum_{j=1}^{n} -s[j]$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{1}{4}q_B + 1$	type 19 	type 20 
$q_B = \frac{1}{n}\sum_{j=1}^{n} -2s[j]$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right)$ $= -\frac{1}{128}q_B^2 + \frac{1}{16}q_B + 1$	type 21 	type 22 
$q_B = \frac{1}{n}\sum_{j=1}^{n} -3s[j] + 7$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$	type 23 	type 24 
$\{\mathscr{W}[1], ..., \mathscr{W}[n]\} \sim \mathcal{N}(-3, 4^2)$ $q_B = \frac{1}{n}\sum_{j=1}^{n} \mathscr{W}[j]s[j] + 7$ $\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$	type 25 	type 26 

Table 3: List of the repayment probability distributions considered in our study with more negative weights for the features.

<table>
<tr><th>features distribution, $\mathbb{P}(S)$</th><th colspan="2">repayment probability, $\mathbb{P}\left(B=1 \mid \hat{S}, M\right)$</th></tr>
<tr><td>

$S \sim \mathcal{N}\left(2, 0.25^2\right)$</td><td>type 27

</td><td>$\mathscr{W}[j] = \frac{5}{99}(j-1)$
$q_B = \frac{1}{n}\sum_{j=1}^{n} \mathscr{W}[j]s[j] - 4$
$\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$</td></tr>
<tr><td>

$S \sim \mathcal{N}\left(6, 1^2\right)$</td><td>type 28

</td><td>$\mathscr{W}[j] = \begin{cases} 1.5 - \frac{7}{495}(j-1), & j \leq \frac{n}{2} \\ 0.1 + \frac{7}{495}\left(j - \frac{n}{2} - 1\right), & j > \frac{n}{2} \end{cases}$
$q_B = \frac{1}{n}\sum_{j=1}^{n} \mathscr{W}[j]s[j] - 4$
$\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$</td></tr>
<tr><td>

$S \sim \left|\mathcal{N}\left(0, 1^2\right)\right|$</td><td>type 29

</td><td>$\mathscr{W}[j] = 20 - \frac{25}{99}(j-1)$
$q_B = \frac{1}{n}\sum_{j=1}^{n} \mathscr{W}[j]s[j] - 4$
$\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$</td></tr>
<tr><td>

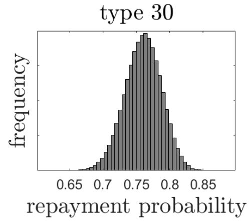
$S \sim \exp\left(\mathcal{N}\left(0, 0.25^2\right)\right)$</td><td>type 30

</td><td>$\mathscr{W}[j] = \frac{10}{99}(j-1)$
$q_B = \frac{1}{n}\sum_{j=1}^{n} \mathscr{W}[j]s[j] - 4$
$\mathbb{P}\left(B=1 \mid \hat{S}, M\right) = \frac{\exp(q_B)}{1+\exp(q_B)}$</td></tr>
</table>

Table 4: List of the repayment probability distributions considered in our study with unbounded features distributions.

for $\mathbb{P}(B = 0 \mid A = 1, S, M) = 1 - \mathbb{P}(B = 1 \mid A = 1, S, M)$. The expectation of the utility given the decision to approve a given application can then be expressed as

$$\mathbb{E}\big[R(\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}) \mid a_{i,t} = 1\big] = m_{i,t}\Big((r+1)\mathbb{P}(b_{i,t} = 1 \mid a_{i,t} = 1, s_{i,t}, m_{i,t}) + e - 1\Big). \tag{91}$$

The algorithm will approve an application when

$$\mathbb{E}\big[R(\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t}) \mid a_{i,t} = 1\big] \geq 0 \tag{92}$$

$$\implies \mathbb{P}(b_{i,t} = 1 \mid a_{i,t} = 1, s_{i,t}, m_{i,t}) \geq \frac{1-e}{1+r}. \tag{93}$$

Thus, in this perfect information scenario, the decision rule can be written as

$$A = \begin{cases} 1 & \text{if } \mathbb{P}(B = 1 \mid A = 1, S, M) \geq \dfrac{1-e}{1+r} \\ 0 & \text{otherwise.} \end{cases} \tag{94}$$

- **Credit Score Based Method.** In real life scenario, we cannot access the true value for the repayment probability $\mathbb{P}(B = 1 \mid A = 1, S)$. Here, we will predict its value by finding the best fit model for the data from the previous lending period. Specifically, we fit the data into the first order Gaussian model,

$$\hat{\mathbb{P}}(B = 1 \mid A = 1, \hat{S}, M) = a_g \exp\left(-\left(\frac{q_g - b_g}{c_g}\right)^2\right), \tag{95}$$

where

$$q_g = \frac{1}{n}\sum_{j \in U(\hat{s})} \hat{s}[j]. \tag{96}$$

Here, $\hat{\mathbb{P}}(B = 1 \mid A = 1, \hat{S})$ is the predicted repayment probability parameterized by constants $a_g,\ b_g,$ and c_g. The constants $a_g,\ b_g,$ and c_g are obtained by minimizing the non-linear least square of the difference between the predicted and the actual repayment probability, *i.e.*,

$$\{a_g, b_g, c_g\} = \underset{\{a_g, b_g, c_g\}}{\arg\min} \sum \left\|\hat{\mathbb{P}}(B = 1 \mid A = 1, \hat{S}, M) - \mathbb{P}(B = 1 \mid A = 1, S, M)\right\|_2. \tag{97}$$

We can then rewrite the decision rule (94) as

$$A = \begin{cases} 1 & \text{if } \hat{\mathbb{P}}(B = 1 \mid A = 1, \hat{S}, M) \geq \dfrac{1-e}{1+r} \\ 0 & \text{otherwise.} \end{cases} \tag{98}$$

The procedure is executed by employing the `fit()` (The MathWorks 2019) function in MATLAB. For this approach, we assume that the true value for the repayment probability is available at the end of every lending period. The MFI approves all applicant at the first lending period then store the features data and the actual repayment probability at the end of the lending period to be fitted to the model for the next lending period. Then, at the end each lending period after the first, we revise the model by adding more data point using the current data. We repeat the process until the tenth lending period as we observed that there is not significant difference in the performance after the tenth lending period.

- **Perceptron.** Perceptron (Rosenblatt 1957) is a classic learning algorithm for binary classification (*i.e.*, approve or decline applications). Specific perceptron algorithm we implemented in our simulation can be seen in Algorithm 2.
- **Random Forest.** We employ MATLAB built-in function for decision tree classification, `fitctree()` (The MathWorks 2019), to predict if the applicants will return or default the loan. We let the function to optimize its hyper-parameters automatically and all other parameters utilize its default values. The decision tree model was trained at every loan period using the available feature information $\hat{S}$ and the boolean return/default data collected from previous lending periods. Here, at the first lending period, the MFI will approve all applicants then store the data at the end of the lending period to be used as the training data for the next lending period. Then, at the end of the next lending period, we add the new data into the training data. To save computational cost from training the model, we perform the training only until the tenth lending period.
- **Support Vector Machine.** For the support vector machine (SVM) classification, because the high dimensionality of the feature vector, we employ binary linear classifier available as a MATLAB built-in function, `fitclinear()` (The MathWorks 2019), with `'svm'` as the `'Learner'` option. Similarly as before, the hyper-parameters of the function are automatically optimized and all other parameters are set to use its default values. The model was trained using the same procedure as the above random forest classification.
- **Logistic Regression.** Lastly, we compare the performance of our proposed algorithm against the logistic regression method. We again employ MATLAB built-in function, `fitclinear()` (The MathWorks 2019), to predict if the applicants will return or default the loan. Here, the `'logistic'` is chosen as the `'Learner'` option. All other settings and procedures are equal to the settings and method used in the above SVM classification.

Algorithm 2: Perceptron

Initialized the perceptron weights P_w and bias P_b
for each loan period t **do**
  for each application i **do**
    Set all empty entry as 0.
    Calculate the perceptron activation $P_A = \sum_{j=1}^{n} P_w[j]s_i[j] + P_b$
    Generate the decision of application i with: $a_{i,t} = \begin{cases} 1; & P_A > 0, \\ 0; & P_A \leq 0. \end{cases}$
    Observe outcome $b_{i,t} \in \{0,1\}$.
    Gain utility $R(\hat{s}_{i,t}, m_{i,t}, a_{i,t}, b_{i,t})$.
    if $a_{i,t} \neq b_{i,t}$ **then**
      Update $P_w = P_w + b_{i,t}s_{i,t}$.
      Update $P_b = P_b + b_{i,t}$.
    end if
  end for
end for

Multi-optimization for improved learning speed We initiated the optimization scheme with 10 initial random points and kept five of the best result for the next lending period. To explore the landscape, we then chose additional five random points for the next iteration and performed the optimization iteration for these 10 points. To reduce the computational cost of the multi-optimization scheme, we only repeated this procedure for the first 50 lending period and then only performed the gradient optimization for the best result afterward.

Step size, learning speed, and convergence The choice of step size α_t is crucial for the convergence and learning speed of the proposed algorithm. We studied the effect of different choices of α_t in our proposed algorithm by varying the constant value of $\frac{D}{G} \in \{0.01, 0.05, 0.1, 0.5, 1, 5, 10, 50, 100, 500, 1000, 5000, 10000, 50000\}$ from Corollary 4.2. A small step size will converge to non-optimum results, while a large step size will produce an overshoot and hinder convergence as can be seen in Figure 3. Corollary 4.2 suggest decreasing the step size with speed $O(1/\sqrt{t})$ which prevents the algorithm to overshoot when the optimum policy has been found.

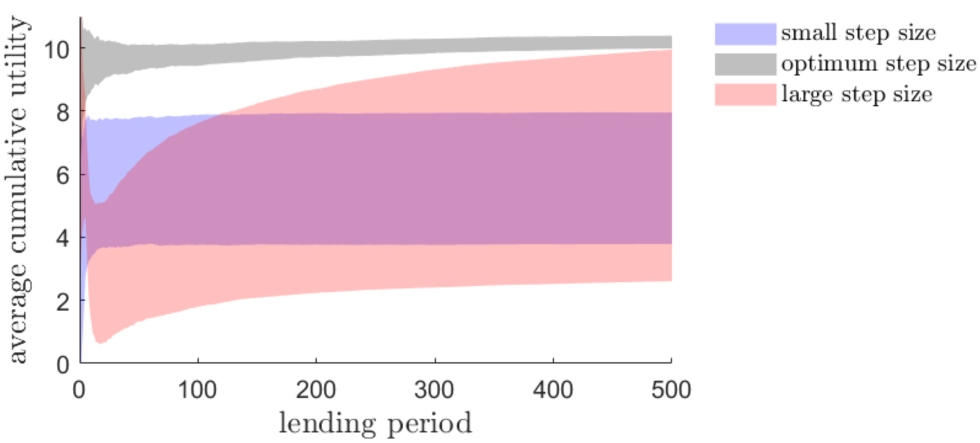


Figure 3: Convergence rate comparison of using different step sizes. For this comparison we consider case A as the form of $L(q)$ with repayment probability distribution type 5 without missing information. We run the simulation 50 times and the shaded regions show the area within one standard deviation of the average cumulative utilities. For this particular case, the optimal step size is achieved when $\alpha_t = \frac{10}{\sqrt{t}}$.

E Performance

Performance Comparison

The performance comparison of the proposed algorithm against other approaches described previously, with $e = 0$ and repayment probability distribution type 5, can be seen in Figure 4. For this comparison, we chose the case's optimum step size and performed

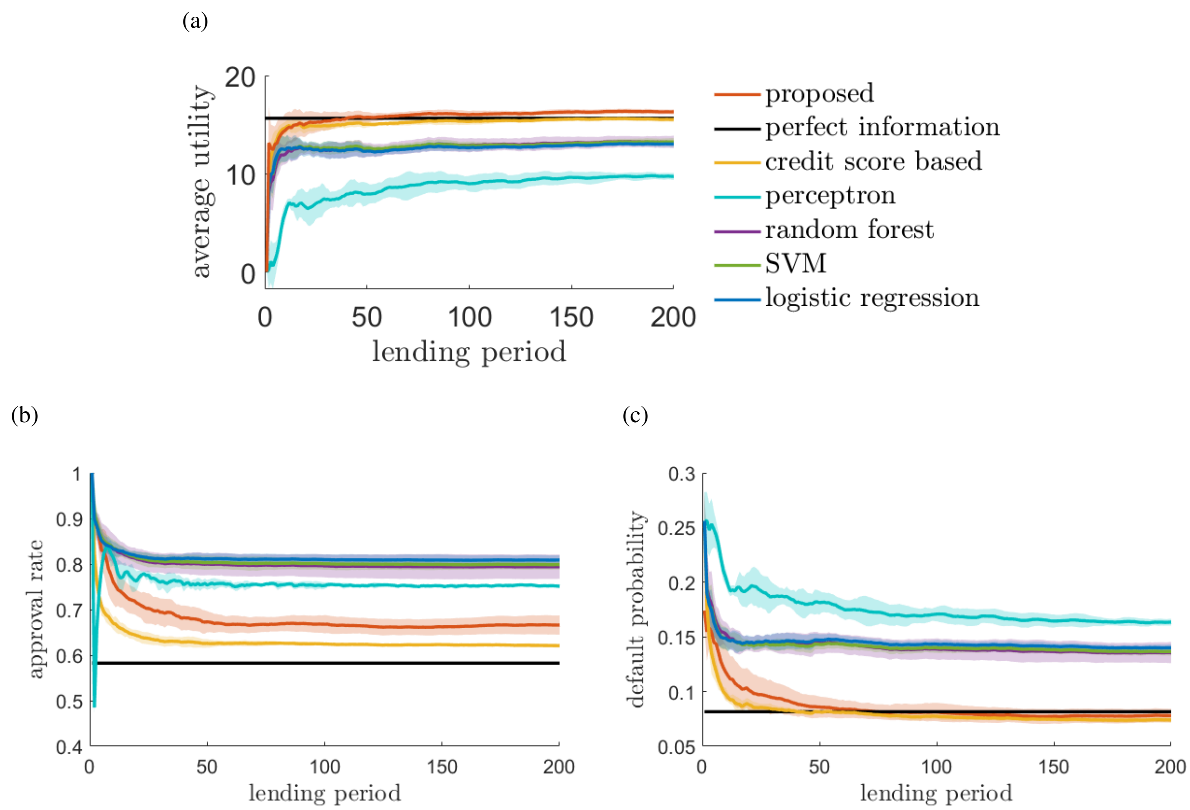


Figure 4: Comparison of (a) average cumulative utility, (b) approval rate, and (c) default probability of the proposed algorithm against several common learning algorithms for binary classification without financial inclusion. The simulation was run 50 times where the solid lines are the average values and the shaded areas show the values within one standard deviation.

the simulations 50 times. Figure 4a shows the comparison of the average cumulative reward defined as

$$V = \frac{1}{t}\sum_{\tau=1}^{t}\sum_{i=1}^{N_\tau} R(\hat{s}_{i,\tau}, m_{i,\tau}, a_{i,\tau}, b_{i,\tau}). \tag{99}$$

We can see that the proposed algorithm gradually increases its average cumulative utility with every iteration. Here, we can see that the proposed algorithm outperforms the other algorithms and is able to approach the performance of the perfect repayment information scenario with a faster convergence rate, at least for this type of repayment probability distribution. The performance comparison with other repayment probability distributions and the Kiva data set will be discussed further. Figures 4b and 4c show the approval rate and the default probability of the compared methods, where we can see that the proposed algorithm converges at a similar approval rate and default probability as the perfect information scenario which shows promising performance of the proposed algorithm.

Robustness to Missing Data

We simulate missing information by choosing a missing probability p_e. Here, the feature information $s_{i,t}$ will still have all information, but the corresponding observe entry in $\hat{s}_{i,t}$ will be empty with probability p_e. The repayment probability of each applicant is calculated without any missing information. We varied the missing probability equal to 0, 10%, 25%, and 50%.

Adaptation to Changes

We inspected the adaptability of the algorithms to a change in the application distribution by introducing different distributions at lending period 250 where the algorithms do not have prior knowledge of if and when the distribution has changed. To highlight the behavior of the algorithms in adapting to the changes, we chose the distributions that will have similar repayment probability distributions but need to have opposite feature weights. Specifically, we chose the following six cases:

1. distribution type 1 changes to distribution type 19,
2. distribution type 2 changes to distribution type 20,
3. distribution type 3 changes to similar distribution but with $\mathbb{P}\left(B = 1 \mid \hat{S}, M\right) = -\frac{1}{4}q_B + 1$,
4. distribution type 9 changes to similar distribution but with $q_B = \frac{1}{n}\sum_{j=1}^{n} -s[j] + 4$,
5. distribution type 27 changes to the same distribution but with $S \sim -\left(\mathcal{N}(2, 0.25^2) - 2\right) + 2$,
6. distribution type 28 changes to the same distribution but with $S \sim -\left(\mathcal{N}(6, 1^2) - 6\right) + 6$,

The distribution types referred to can be found in Appendix D. We trained the credit score based method, random forest, SVM, and logistic regression algorithms for 10 lending periods in the beginning but keep storing the data at the end of every lending period. When the algorithms notice a distribution change, implies by the sudden drop of the utility, the algorithms will start the training again with the collected data until converge.

Figure 5 shows the statistical summary of the convergence utility and rise time of each algorithm before and after the distribution changes. As shown, all algorithms are able to recover their convergence utilities after the change. However, as expected, the algorithms designed for offline use such as the credit score based method, random forest, SVM, and logistic regression require a longer time to recover after the change. This is because these algorithms, unlike the proposed method and the perceptron which can use immediate feedback from the latest data, end up using samples that contain both data from before and after the change to update the model.

Performance Against Different Distributions

We compared the performance of the algorithms against different distributions described in Appendix D. We run the simulation 50 times and take the average of the converged utility values of each distribution type. To fairly compare all different type of distributions, we normalized the utilities by shifting it up such that the lowest converge utility become zero. Then, we divided the shifted value of the utilities by the utility of the perfect scenario after shifting and multiply it by two. Finally, we shifted the values down by one. Here, we considered the individual lending case where there are 10% missing information for all 30 artificial distributions and 20% missing information for the data inspired by the Kiva dataset.

The final average normalized utilities from the 18 different distributions listed in Table 2 obtained by each algorithm can be seen in Figure 6 with its rise time is shown in Figure 6b. Here, we can see that our proposed method provides higher converged utilities with lower rise time at most of the considered distributions. The proposed method still maintain consistent performance against repayment probability distributions with more negative weight of the features such as the distribution types 19 to 26 described in Table 3, as can be seen in Figure 7. From Figure 7 we can also see that in average the proposed algorithm gives higher final utility with lower rise time than the other compared algorithms for unbounded applicants' feature distribution such as distribution types 27 to 30 describe in Table 4. The statistical summary of the result can be seen in Figures 2c and 2f.

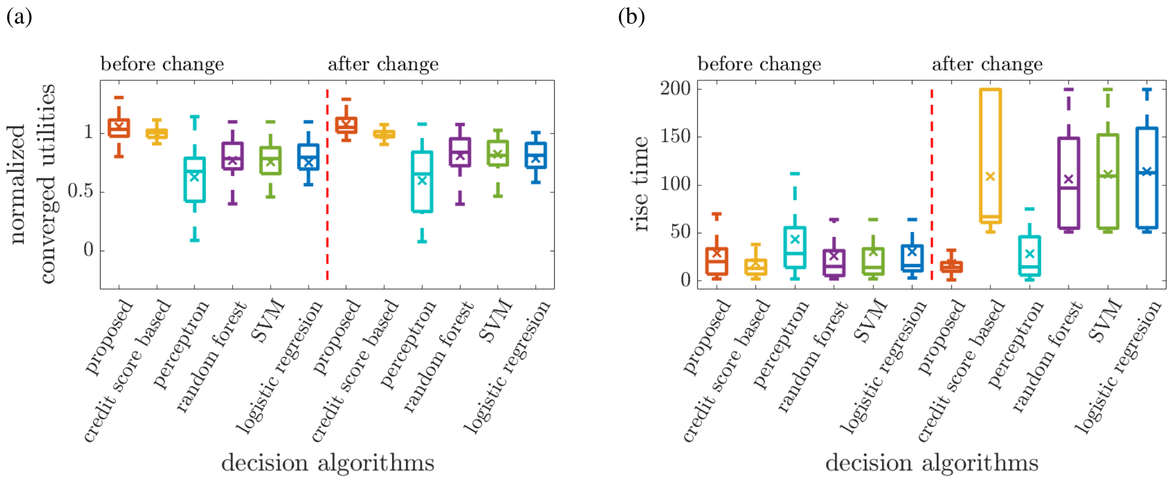


Figure 5: Statistical summary of the (a) convergence utilities and (b) rise time of each algorithm before and after the distribution changes. The mean values are shown by '$\times$', and the first, second, and third quartiles as well as the maximum and minimum values are shown as the boxplot. The rise time after the change is calculated from the time the change takes place.

From the statistical summary, in average, our proposed approach converged to the optimal utilities faster with relatively small uncertainty. In addition, considering dataset generated based on the real lending data from Kiva.org, the proposed algorithm able to give comparable performance against the other algorithms as showed in Figure 8. Moreover, from Figure 8 we can see that the proposed algorithm results in higher utility and lower rise time than most of the other algorithms when there are 20% missing information. These results also show the robustness of the proposed approach against different repayment probability distributions.

Group Dynamics

As mentioned, we considered two cases of group lending strategies as described in Appendix D. For the basic group lending that takes group size as the only feature, Figure 9a shows the theoretical relationship between expected reward and group size. The optimal threshold is shown as the cross-over point across 0. From Figure 9a, the optimal threshold is around $n = 20$. The curve increases monotonically after it goes to the positive domain, implying that the larger group size gives a greater expected reward. For the advanced group lending, where the group size is part of the applications' features, Figure 10 shows the final average normalized utilities and its rise time from the 18 different distributions listed in Table 2 with advanced group lending adaptation. Here, we considered 10% missing information and applied the same normalization treatment as described in individual lending case. From the results we can see that the proposed algorithm also able to provide higher final utility with comparable rise time compared to the other algorithms.

Fairness Investigation

To investigate the fairness of our algorithm, we introduce a supposedly discriminative feature to the feature vector. This discriminative feature has three discrete values $\{0,2,4\}$ and does not affect the actual repayment probability. The algorithm is then run for some target ratio of the discriminative feature with a value of 0 to investigate the type 2 fairness. To investigate type 3 fairness, we consider the discriminative feature with values 0 and 4.

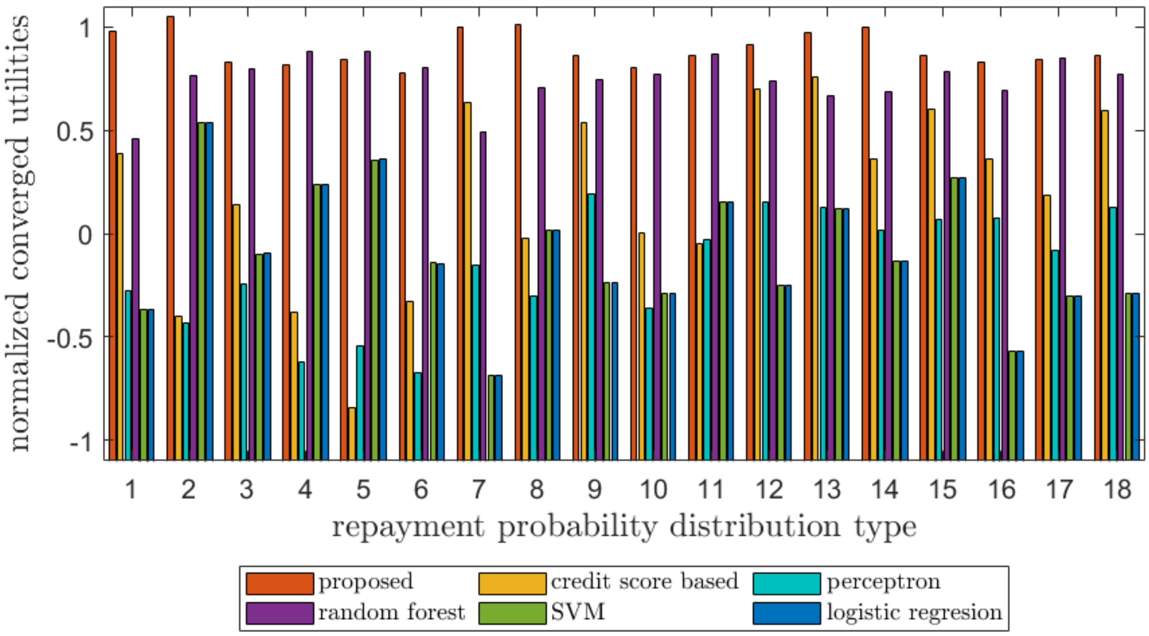
Effect of Financial Inclusion

Here, we varied the loan subsidy level from 0 to 1 with 0.05 interval. We then record the final approval and default rates of each value of e. From Figure 11 we can see that the approval and default rates of the proposed algorithm changes as we change the value of e, suggesting that we can control the approval vs default tradeoff by altering the loan subsidy level. As we can also see that the perceptron, random forest, SVM, and logistic regression algorithms do not posses the flexibility to optimize the tradeoff between approval vs default rates.

Computational Cost Comparison

We compared the computational time of the algorithms by varying the number of features information (see Figure 12a). From the comparison, we can see that the computational time for all algorithm is almost stable for different number of features information. Figure 12b shows the statistical summary for the computational time of each algorithm with 100 features information. From

(a)



(b)

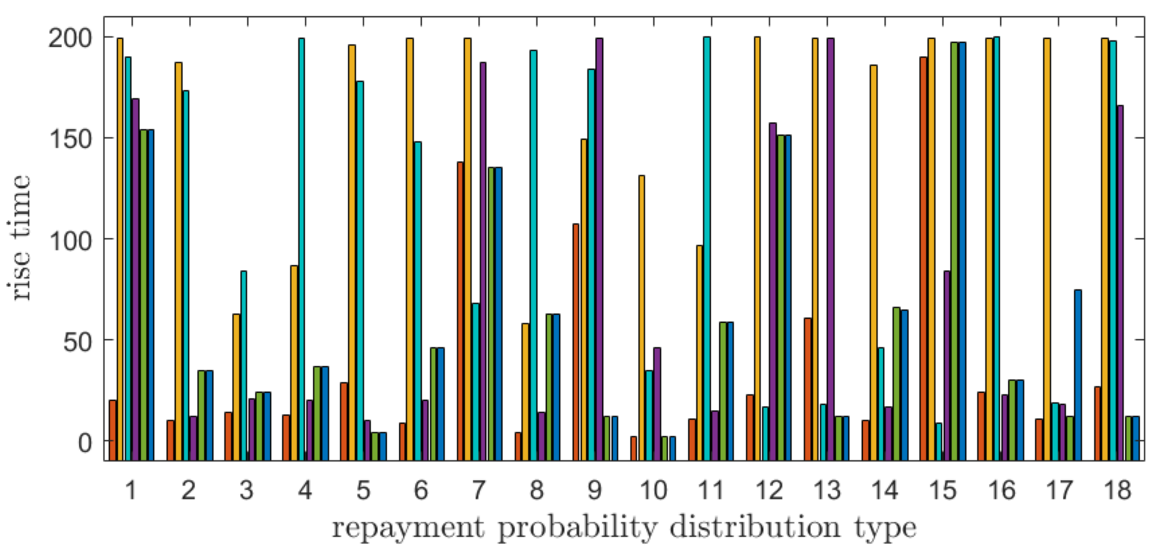
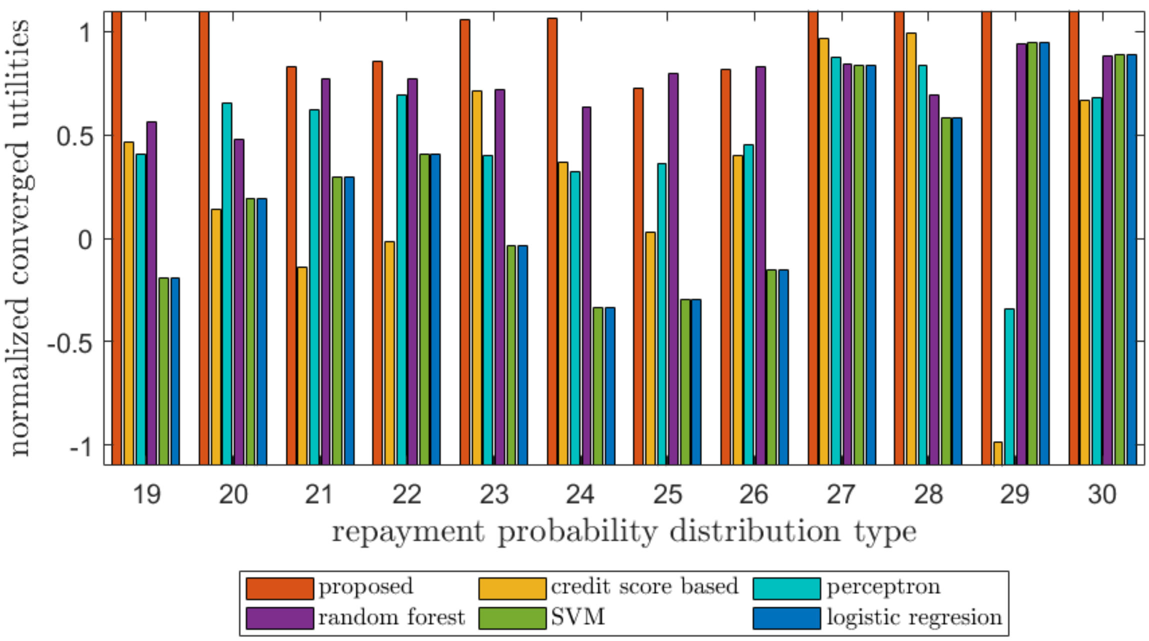


Figure 6: Considering the case with 10% missing information for distributions type 1 to 18 in Table 2. (a) shows bar plot of the converged utilities from different algorithms normalized by the converged utility from the perfect repayment information scenario. (b) shows bar plot comparing the rise time of the algorithms.

(a)



(b)

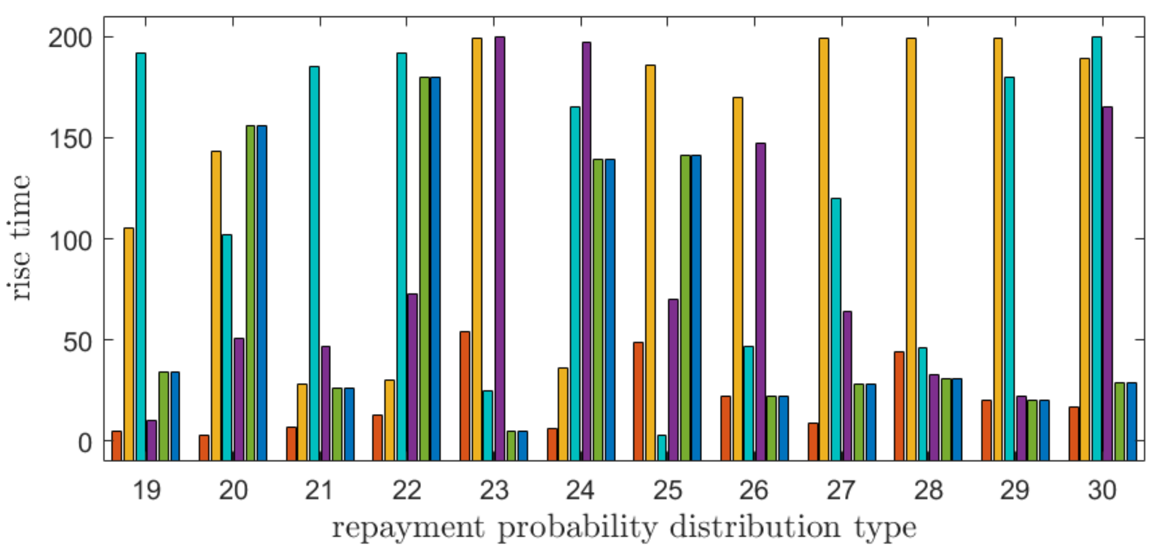
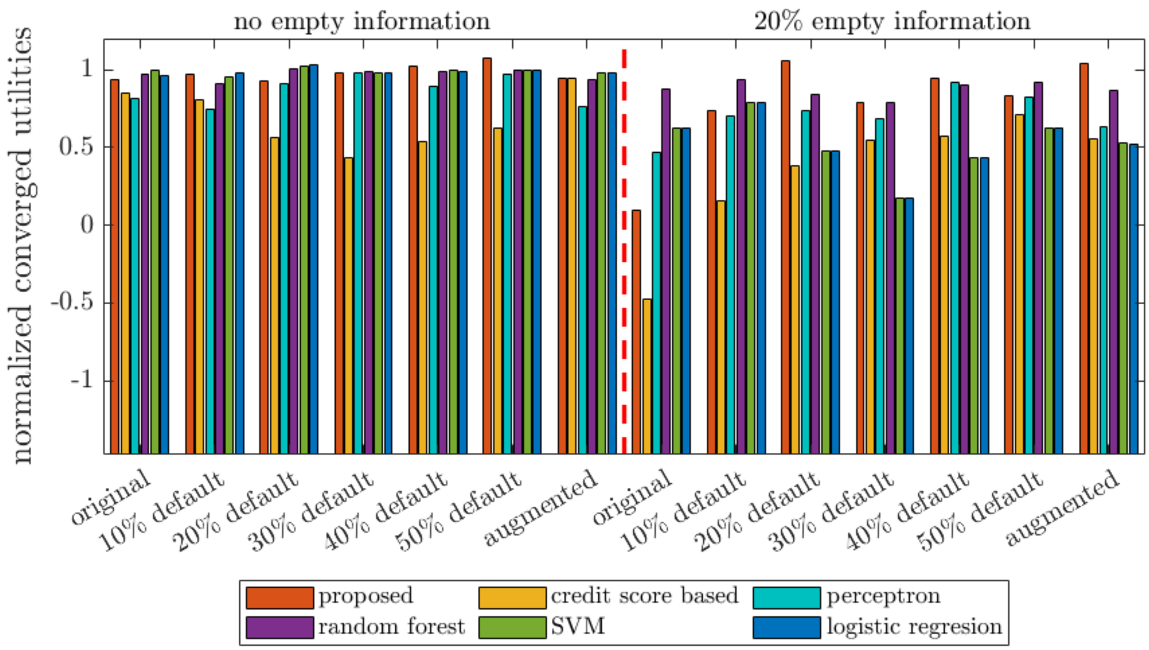


Figure 7: Considering the case with 10% missing information for distributions type 19 to 30 in Tables 3 and 4. (a) shows bar plot of the converged utilities from different algorithms normalized by the converged utility from the perfect repayment information scenario. (b) is bar plot comparing the rise time of the algorithms.

(a)



(b)

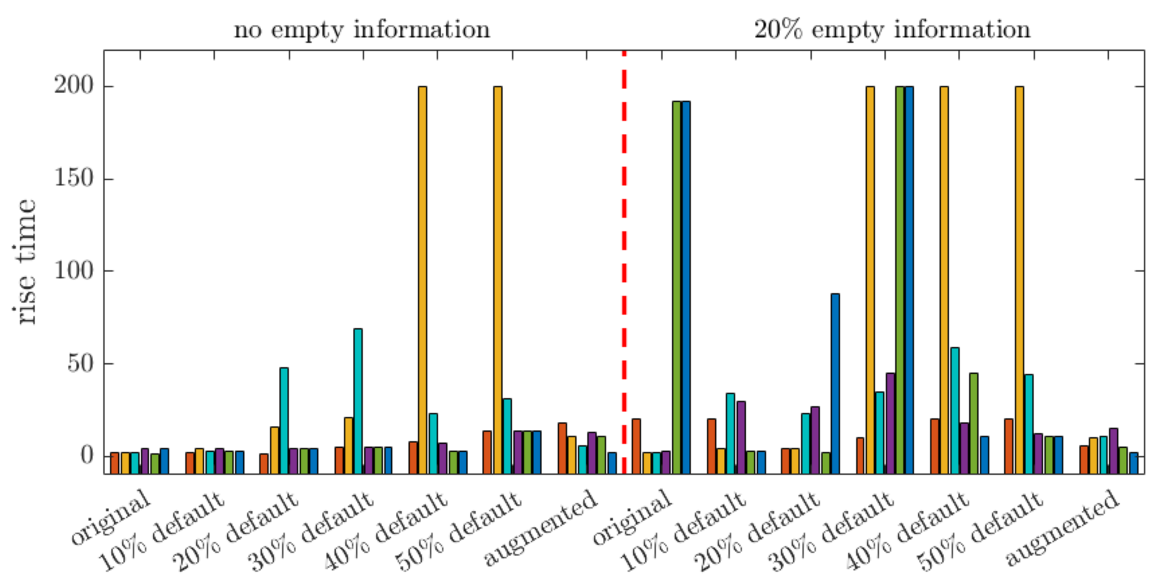


Figure 8: Considering dataset from kiva.org. (a) shows bar plot of the converged utilities from different algorithms normalized by the converged utility from the perfect repayment information scenario. (b) shows bar plot comparing the convergence time of the algorithms.

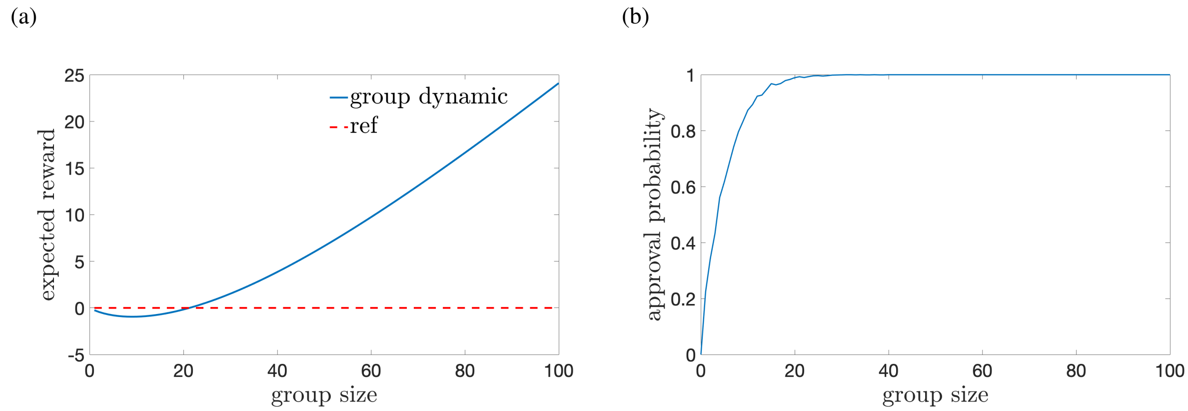
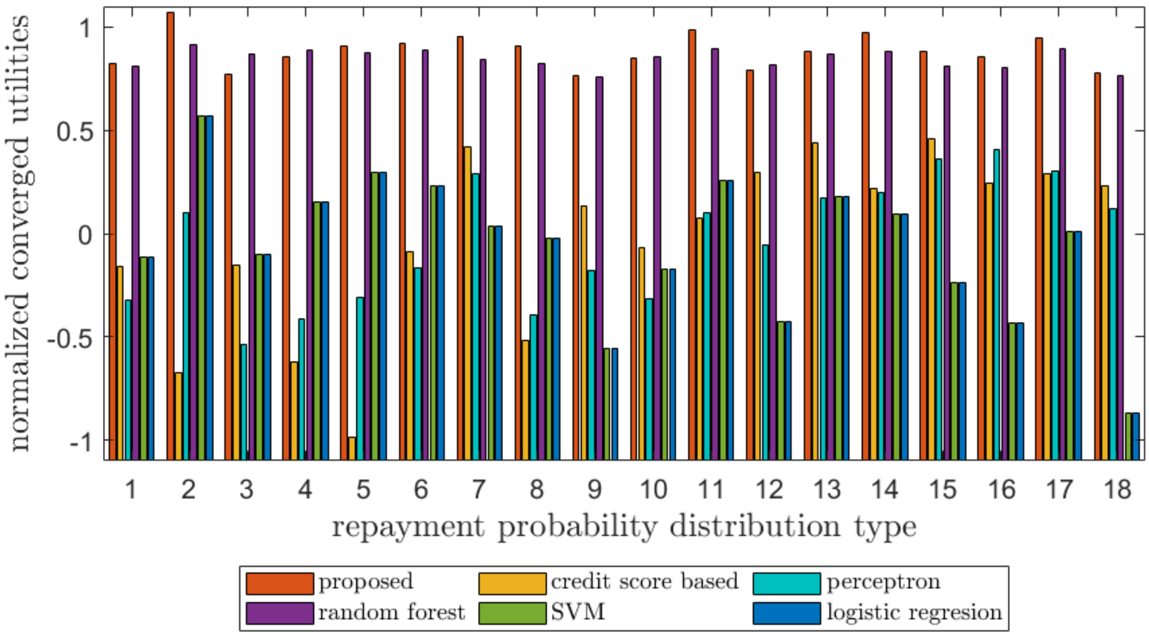


Figure 9: (a) Theoretical study on optimal group threshold: optimal threshold n is around 20. (b) Empirical result on optimal group threshold: optimal threshold n is around 20.

Figure 12, we can see that the proposed algorithm is computationally cheaper in order of magnitude compared to all other algorithms except the perceptron. However, although the perceptron is computationally cheaper than the proposed algorithm, the proposed algorithm achieves better performance and comparable to the other more computationally expensive algorithm such as the random forest algorithm.

(a)



(b)

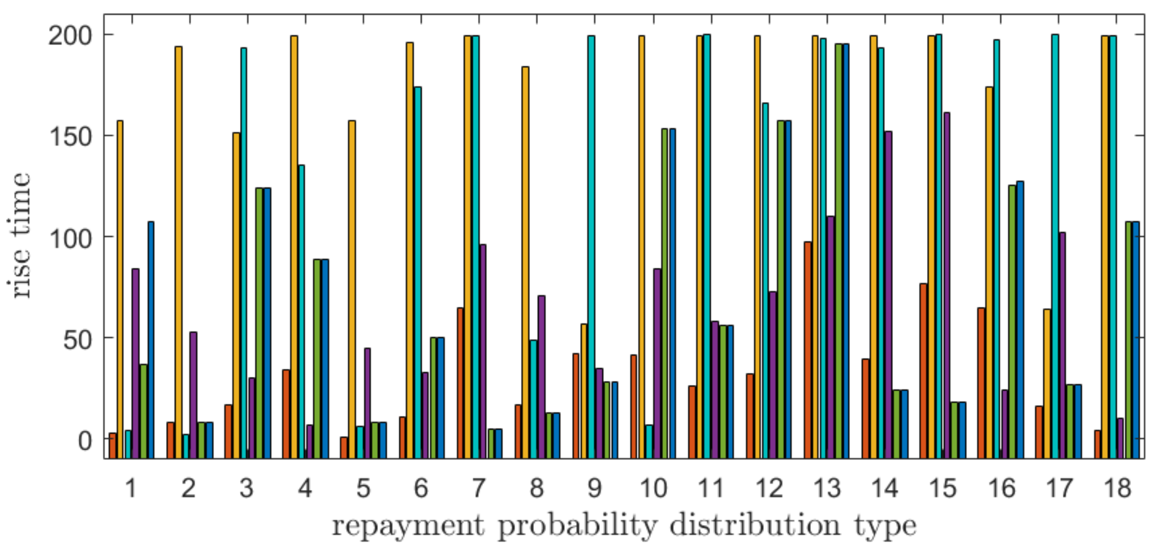


Figure 10: Considering the advance group lending scenario with 10% missing information for distributions type 1 to 18 in Table 2. (a) shows bar plot of the converged utilities from different algorithms normalized by the converged utility from the perfect repayment information scenario. (b) shows bar plot comparing the rise time of the algorithms.

(a) (b)

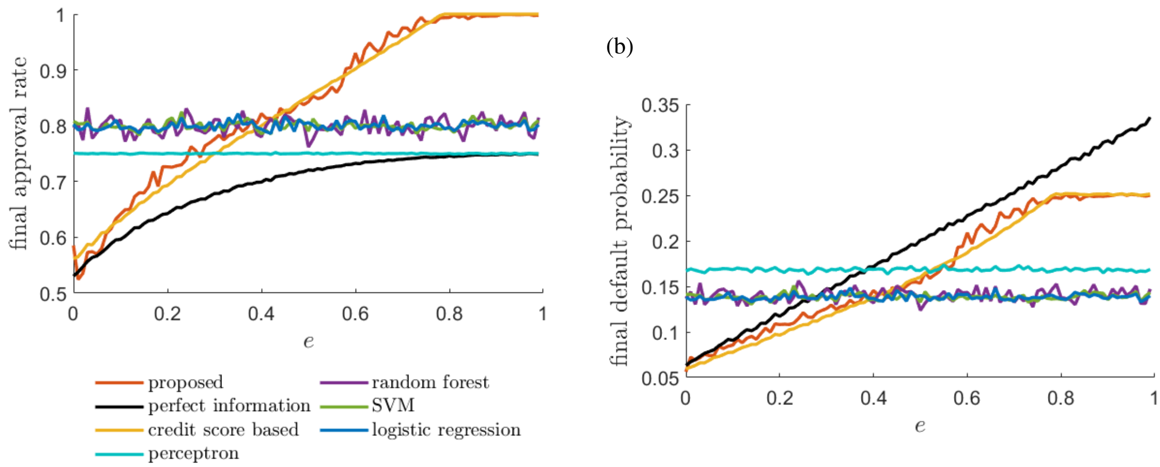


Figure 11: Plot showing the effect of the constant e in (34) as the subsidy level received by financial institution to (a) the loan approval rate and (b) default rate.

(a) (b)

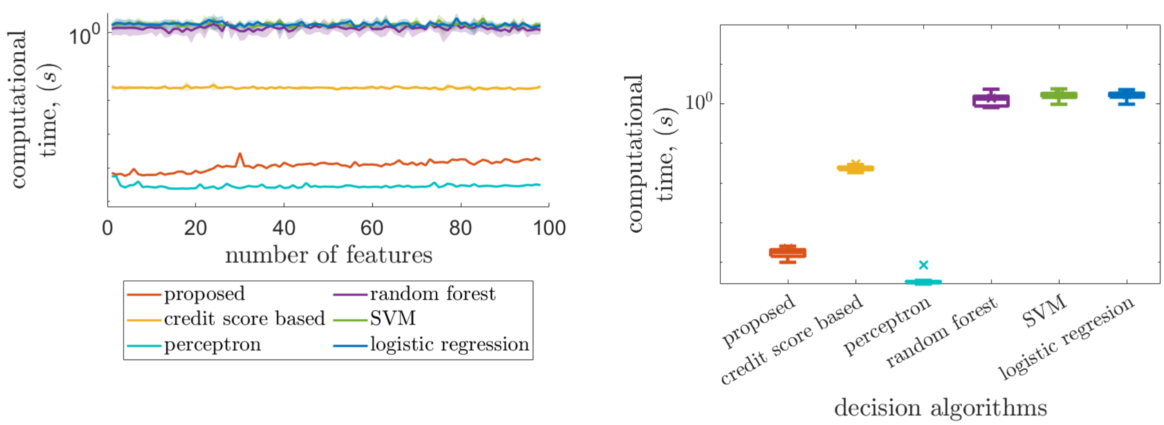


Figure 12: (a) shows computational time comparison of different algorithms for increasing number of feature information. (b) shows computational time comparison for 100 features information. The plots are generated from type 5 distribution. The vertical axes of both plots are in logarithmic scale.

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